PROBLEM 1. Is it possible to solve the equation

\[ z^3 + 3xyz^2 - 5x^2y^2z + 14 = 0 \]

for \( z \) near 2 as a function of \( x \) and \( y \) near \( x = 1, y = -1 \)? Explain your reasoning.

PROBLEM 2. Let \( G : \mathbb{R}^2 \to \mathbb{R} \) be a \( C^1 \)-function. State under what conditions does the equation \( G(x, y) = 0 \) define \( y \) near some \( b \in \mathbb{R} \) as a differentiable function in \( x \) near some \( x = a \in \mathbb{R} \). How can the derivative \( \frac{dy}{dx} \) be expressed near \( a \)?

PROBLEM 3. Simplify the differential form

\[ \varphi = (x^2 \, dx \wedge dy - \cos x \, dy \wedge dz) \wedge (y^2 \, dy + \cos x \, dw) - (x^3 \, dy \wedge dz - \sin x \, dy \wedge dw) \wedge (y^3 \, dy + \sin x \, dz). \]
PROBLEM 4. Apply Leibniz’s formula to evaluate the exterior derivative of the differential form 
\[ \omega = (e^{xy} \, dz + e^{yz} \, dx) \wedge (\sin x \, dy + \cos y \, dx). \]

PROBLEM 5. Prove that if \( \varphi \) is an \( r \)-form with \( r \)-odd, then \( \varphi^2 = 0 \).

PROBLEM 6. Find the exterior derivative of the differential form \( \varphi = 3xy \, dx + 2x \, dy \). Is \( \varphi \) a closed form or not? Use the result to show that one cannot find a function \( f \) such that \( \varphi = df \).

PROBLEM 7. Consider the surface \( S \) of equation \( z = 3x^2 + 5y^2 \).

(U) Write the equation of the tangent plane to \( S \) at the point \( p = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \).

(G) Write the equations of the tangent planes \( P_1, P_2, P_3 \) to \( S \) at the points \( p_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, p_2 = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}, p_3 = \begin{pmatrix} 0 \\ 2 \\ 20 \end{pmatrix} \). Find the point \( q = P_1 \cap P_2 \cap P_3 \).
PROBLEM 8. Let $U = \left\{ \begin{pmatrix} u \\ v \\ w \end{pmatrix} : 0 \leq u, v, w \leq 1 \right\}$, $\gamma : \mathbb{R}^3 \to \mathbb{R}^4$ is the function

$$\gamma \left( \begin{pmatrix} u \\ v \\ w \end{pmatrix} \right) = \begin{pmatrix} uv \\ u^2 - w^2 \\ v - u \\ w \end{pmatrix},$$

and $M$ is the manifold $\gamma(U) \subset \mathbb{R}^4$. Express the integral

$$\int_{[\gamma(U)]} x_2 \, dx_1 \wedge dx_3 \wedge dx_4$$

as an ordinary integral in $\mathbb{R}^3$. (U) Do not evaluate it. (G) Evaluate it.

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PROBLEM 9. Why does the form $dx$ not define an orientation of the unit circle $x^2 + y^2 = 1$?

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PROBLEM 10. Write down, in your own words, all ways you know in which a $k$-dimensional manifold $M \subset \mathbb{R}$ can be described locally near one of its points, say, $c \in M$. 

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