FOR THE REST OF YOUR LIFE

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'For the Rest of Your Life' is a new TV game show. Contestants play to win money every month. This can be for as little as one month or, if every one of their guesses is correct, for the rest of their lives. The rules are shown in table 1.

First half of the programme
Contestants are faced with 11 tubes. Eight of these tubes have a white light inside and three have a red light. The contestant chooses a tube at random. Picking a white light increases their prize by £150, picking a red lowers it by the same amount. Once they are four steps up the money ladder they can stick with the prize they have. i.e. once they have £600 they can stop.
As an example, consider if the tubes chosen were white, white, white, red, white, red white, white. In this case the contestant's possible prizes would have been £150, £300, £450, £300, £450, £300, £450, £600. At this point the contestant is allowed to stop guessing and take the £600 since this is the fourth step up the ladder. In this case it may well be worthwhile to do so because once all three reds have been picked the contestant wins nothing.

Second half of the programme

Contestants are faced with 15 tubes. Eleven of these tubes have a white light inside and four have a red light. The lights now count for months for which the money won in the first half of the programme is paid. The possibilities are 1 month, 3 months, 6 months, 1 year, 2 years, 3 years, 5 years, 10 years, 15 years, 25 years and 'the rest of your life'. (The 'rest of your life' is taken as 40 years.)
If a contestant drew all 11 white lights without drawing a red light then he/she would win an amount of money (won in the first half of the programme) every month 'for the rest of their life'.

Table 1

The first problem that will be analysed is: **In the first part of the programme, if the contestant stops as soon as he/she has £600, how likely is it that he/she will win £600?**

The scenarios that will win £600 are the tubes being drawn in the following orders.

1) WWWW Probability = 8/11 x 7/10 x 6/9 x 5/8 = 7/33

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2) WWWRWW Probability = 4 x (8/11 x 7/10 x 6/9 x 3/8 x 5/7 x 4/6) = 8/33
3) WWRWWW
4) WRWWWW
5) RWWWWW

6) WWWRRWWW Probability = 14 x (8/11 x 7/10 x 6/9 x 3/8 x 2/7 x 5/6 x 4/5 x 3/4)
7) WWWRWRWW = 14/55
8) WWRWWRWW
9) WWRWRWWW
10) WWRRWWWW
11) WRWWWRWW
12) WRWWRWWW
13) WRWRWWWW
14) WRRWWWWW
15) RRWWWWWW
16) RWRWWWWW
17) RWWRWWWW
18) RWWWRWWW
19) RWWWWRWW

Probability £600 is won = 7/33 + 8/33 + 14/55 = 117/165

The contestant has a good chance of winning £600 - approximately ¾
Suppose, however, he/she decides to try for £750.

The second problem that will be analysed is**: In the first part of the programme, if the contestant stops as soon as he/she has £750, how likely is it that £750 will be won and is it worth trying for £750?**

The scenarios that will win £750 are the tubes being drawn in the following orders.

1) WWWWW Probability = 8/11 x 7/10 x 6/9 x 5/8 x 4/7 = 4/33
2) WWWWRWW Probability = 5 x (8/11 x 7/10 x 6/9 x 5/8 x 3/7 x 4/6 x 3/5)
3) WWWRWWW = 2/11
4) WWRWWWW
5) WRWWWWW
6) RWWWWWW
7) WWWWRRWWW Probability = 20 x (8/11 x 7/10 x 6/9 x 5/8 x 3/7 x 2/6 x 4/5 x ¾ x 2/3)
8) WWWWRWRWW = 8/33
9) WWWRRWWWW
10) WWWRWRWWW
11) WWWRWWRWW
12) WWRRWWWWW
13) WWRWRWWWW
14) WWRWWRWWW

15) WWRWWWRWW
16) WRRWWWWWW
17) WRWRWWWWW
18) WRWWRWWWW
19) WRWWWRWWW
20) WRWWWWRWW
21) RRWWWWWWW
22) RWRWWWWWW
23) RWWRWWWWW
24) RWWWRWWWW
25) RWWWWRWWW
26) RWWWWWRWW

Probability of winning £750 = 4/33 + 2/11 + 8/33 = 6/11
Interestingly, the contestant has a reasonable chance of winning £750
If we look at the expected winnings, however, we see that the contestant is better off trying for £600

Expected winnings, given that he/she is trying for £600, = £600 x 117/165 = £425
Expected winnings, given that he/she is trying for £750, = £750 x 6/11 = £409

Let us suppose that the contestant takes £600 into the second part of the programme.
What strategy should the contestant use to maximise their expected winnings?
Suppose the contestant tried to win £600 a month for the rest of his/her life. The probability of pulling out 11 white lights in succession is
11/15 x 10/14 x 9/13 x 8/12 x 7/11 x 6/10 x 5/9 x 4/8 x 3/7 x 2/6 x1/5 = 11! 4!/15! = 1/1365
clearly it is not in his/her interest to try and win the money for the rest of his/her life!

In fact most contestants adopt a strategy of playing until only one red light remains.
This strategy will be analysed.
What are the expected winnings of a contestant who plays until three red lights have been revealed?

(In fact it may be the case that the contestant who adopts this strategy never actually sees three red lights – all the white lights may show before three red lights are revealed.)
The outcomes, their associated probabilities and winnings are shown in table 2.

Outcome	**Probability (=p)**	**Winnings (=W)**	**pW**
11 white	1/1365	£600x480	£600x480/1365
11 white, 1 red	11/1365	£600x300	£600x300x11/1365
11 white, 2 red	66/1365	£600x180	£600x180x66/1365
11 white, 3 red	0	0	0
10 white, 3 red	132/1365	£600x60	£600x60x132/1365
9 white, 3 red	165/1365	£600x36	£600x36x165/1365
8 white, 3 red	180/1365	£600x24	£600x24x180/1365
7 white, 3 red	180/1365	£600x12	£600x12x180/1365
6 white, 3 red	168/1365	£600x6	£600x6x168/1365

5 white, 3 red	147/ 1365	£600x3	£600x3x147/1365
4 white, 3 red	120/1365	£600x1	£600x1x120/1365
3 white, 3 red	90/1365	0	0
2 white, 3 red	60/1365	0	0
1 white, 3 red	33/1365	0	0
3 red	12/1365	0	0

Table 2

(Note that it is not possible to pull 11 white lights and 3 red lights using this strategy.
Since the last light pulled is red the 11th white light will have been pulled previously.
The contestant stops pulling once the 11th white light has been pulled and hence it is not possible to pull 11 white lights and 3 red lights.)

(To see how these probabilities are calculated consider the probability of 5 white and 3 red

$= \mathbf{(}11/15 \times 10/14 \times 9/13 \times 8/12 \times 7/11\mathbf{)} \times \mathbf{(}4/10 \times 3/9\mathbf{)} \quad \times \mathbf{(}2/8\mathbf{)} \quad \times \mathbf{(}7!/(5!2!)\mathbf{)}$

5 white ***2 red*** ***last one red*** ***number of combinations of 5 white and 2 red***

$$= \frac{11!\, 4!\, 7!\, 7!}{6!\, 15!\, 5!\, 2!}$$

(Note that the last tube picked has to be red because the contestant stops pulling once 3 red lights show)

In general the probability of X white and 3 red is $\frac{11!\, 4!\, (15 - (X+3))!\, (X + 2)!/(X!\, 2!)}{15!\, (11 - X)!}$

where X< 11)

The expected winnings are Σ pW = £16513.
Not bad winnings for pulling lights out of a tube at random!!