HOW TO INCREASE MATHEMATICAL CREATIVITY- AN EXPERIMENT

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Abstract: Creativity is an integral part of mathematics. In this article I examine the increase in awareness of creativity in mathematics using Fröbel's blocks in a college classroom. A majority of students found the introduction of the "gifts" of the founder of Kindergarten to a college geometry classroom enhancing their interest in mathematics. They judged the wooden blocks helpful in their understanding of geometry. The students showed increased awareness of creativity in mathematics.

Keywords: creativity; Fröbel's blocks; geometry; teaching of geometry; reflective practice

1. Introduction

Many students dislike classes in mathematics. They give a wide variety of reasons for this and among the most mentioned ones are that mathematics is hard, mathematics is boring and mostly irrelevant. Part of this problem stems from misconceptions about mathematics. It is described as inflexible and formulaic as opposed to fun and creative. As a teacher of mathematics it is my duty to counteract those prejudices and create a fertile learning environment. I continually seek to inspire students and convince them that mathematics in all its forms is worthwhile.

In this paper I describe an experiment aimed at revealing the creative process in mathematics. Creativity enters mathematics in many different ways. Three important ways are abstraction, connection, and research. The creativity of abstraction concerns the creation of models that reflect the real world and can be solved with mathematical tools known to the individual. The creativity of connection is the realization that known mathematical tools can be applied to new problems, allowing problems to be viewed in a new way. Connections are also made when mathematical and other knowledge come together to understand and solve problems from a variety of areas. Finally, the creativity of researching is the discovery of new mathematical tools that fit unsolved problems and add to the available tools for other users of mathematics.

The class chosen for this experiment was an undergraduate college class in Euclidean geometry populated by aspiring teachers, and the tool for creative development was Friedrich

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Fröbel's "gifts", based on their simplicity as well as scientific and geometric connections. The students were presented with the gifts and Fröbel's instructions to examine a possible increase in awareness of creativity in mathematics.

1.1. Euclidean Geometry

Euclidean Geometry is the study of plane and space objects. Its main concern is the relations and interactions of points, lines and polygons. Euclid was the first to systematize the study of geometry in his work *Elements*. From basic premises (axioms) and common notions he developed successively more complicated facts. The proof of each new proposition or theorem was based only on the axioms, common notions and previously proven propositions and followed a logical path.

However, high school geometry is mostly presented without proofs, or at most a limited exposition to the logical structure that lies at the foundation of Euclid's *Elements*. As a result, knowledge of basic notations and logical foundations are often lacking in students. Also, three-dimensional geometry often is neglected or omitted. Lastly, the students supposedly discover many facts with tools such as the Geometer's Sketchpad or other applications. While this element of discovery helps the students understand and retain some of the information, it obscures the structures of geometry.

Creativity in the creation of proofs is a fundamental part of mathematics. In geometry there are often several ways of proving a theorem and sometimes the proofs are far from obvious. In addition, students often lack the understanding of why certain statements have to be proven. The most common example is that the interior angle sum in a triangle in the Euclidean plane is 180°. The students all know this fact but when it comes to proving it they often do not know why a proof is required or how to attempt it. Some students lack the creativity to think of situations where the angle sum might be different or to (re-)create a valid proof of the fact. The first case can be remedied by drawing triangles on the outside of a sphere where one can easily draw a triangle with angle sum close to 360°. The other situation requires more work.

1.2. Friedrich Fröbel

Friedrich Fröbel was a German educator, scientist and naturalist who first introduced the concept of "Kindergarten". He was among the first to realize that learning starts at birth and gave the first task of teaching the children to their mother. He developed the idea of Kindergarten as an aid to early learning and developing inquisitive habits. His main work, *The Education of Men* (Menschenerziehung) (Fröbel, 1826/1905) was published in 1826. One of his many ideas is the use of gifts he designed for the students.

Source: http://www.constructiontoys.com/
Gift three consists of eight 1 inch cubes, gift four consists of eight 1□2□½ inch blocks, gift five consists of cubes, triangular prisms from cutting a cube into half or fourth, and gift six consists of 1□1□½, 1□½□½ and 1□2□½ inch blocks

These gifts are sets of wooden blocks and are used to create objects of beauty, nature and the physical environment. We used gifts two through six in class. A note on the word "gift": it is now the standard English word for describing Fröbel's "Gaben" which could be translated from the German as gift. They were not intended to be presents but merely a tool given to the student. The structures of the gifts can be linked to his studies of crystals under Professor Weiss in Berlin (Rubin, 1989). It can be viewed as an early precursor to the use of manipulatives to aid students in learning. The gifts are rooted in Fröbel's work on crystals and exhibit many of the properties that crystallographers use to describe crystalline structures (Rubin, 1989).

Another of Fröbel's ideas is that of guided discovery, which he promotes throughout his works. The teacher's guidance can take several forms. He can steer students towards certain activities. He can limit the choices put before a student and he can guide the student through questions and prompts. It was his intention to teach young children through play and to instill in them the scientific method at an early age. Guided discovery would at a later age take the form of discussion of the results.

Many current ideas in education were previously presented by Fröbel in his landmark book "*The Education of Men*" and other works. He put a great emphasis on student learning that occurred when the students were actively involved and had hands-on material. This was one of the main ideas of the gifts. Students create objects by manipulating blocks and other tangible objects. He also emphasized the importance of outdoor activity and the first Kindergarten had a plot of land for each child to tend, hence children's garden (Kindergarten).

Fröbel saw a teacher more as a guide than a lecturer. He believed that discovery learning is much more fruitful for the children then being taught concepts without a hands-on activity. However, Fröbel stressed that the guidance by the teacher is of the utmost importance and the gifts again reflect that principle. While his first experiment had about 100 blocks the gifts finally came to their current shape as part of the idea of guidance by limiting the choices given to the children.

2. Literature Review

Recent research into the life and work of Friedrich Fröbel focuses on two basic ideas; his influence in the historic context and how his ideas can be applied to the modern school. In the light of increased standardized testing, William Jeynes (Jeynes, 2006) made his case for a Fröbelian approach for schooling in kindergarten and first grade. Jeynes suggests that "a kindergarten curriculum dedicated to developing mind, the spirit and the body" (Jeynes, 2006, p. 1941) should be developed and we agree with his assertion that it can be found in Fröbel's work. John Manning (Manning, 2005) makes a similar point in his call to re-examine Fröbel's life and gifts. He thinks that the ideas can be used as a supplement to testing rather than in its place.

Many in today's education world dismiss Fröbel as a Romantic educator whose child centered view cannot possibly work in the modern school system, especially since his curriculum does not produce immediately measurable results but is based on the education of the whole person, mind, spirit and body. Like so many others, Reese laments that Fröbel was "alternatively obtuse and highly prescriptive" (Reese, 2001, p.15). He admits that "Froebel's followers substantially revised the … gifts". I believe that in order to understand Fröbel one has to go back to the original documents and learn from the idea and manifestation of Kindergarten. "Fröbel was searching for the unity of things, for order" (Reese, 2001, p.3) and this is reflected in his work in general and his gifts in particular. For more on Fröbel read the exceptional book "A Child's Work" by Joachim Liebschner (Liebschner, 2001).

In contrast, other researchers point out the historical importance of Fröbel to different school systems. For example Meike Baader (Baader, 2004) investigates Fröbel's influence on the American system in conjunction with educational theory while Brehony and Valkanova (Brehony & Valkanova, 2006) investigate the influence on the Russian system.

Use of some of Fröbel's ideas in the modern classroom has been suggested before. Geretschlaeger (Geretschlaeger, 1995) has used the ideas of paper folding or origami in his geometry classroom. The activity of paper folding is one of the "occupations" that Fröbel suggested. Occupations are materials and instructions given to the students just like the gifts. But unlike the gifts the occupations are altered in the process. I have used the gifts before but in an introductory course in modern geometry with a focus on abstraction and connections between seemingly unconnected objects and ideas (Brunkalla, 2006).

The research into creativity is, on the other hand, very voluminous. A good overview of mathematics and creativity can be found in Treffinger et al. (Treffinger, Young, Shelby & Shepardson, 2002). Most research is centered on children from Pre-Kindergarten through grade nine. Few publications deal with creativity in highly accomplished mathematicians. Moreover, there is a curious lack of research in the area of creativity in college mathematics. The most basic problem is that there is no universally accepted definition of mathematical creativity (Haylock, 1997) and no single test or assessment of it. Many researchers agree on certain qualities of creativity but show some divergence on others. Significantly, most researchers link mathematical creativity to mathematical ability. Often a positive attitude towards mathematics is linked to creativity while a negative attitude would imply less mathematical creativity (Mann, 2005).

Another focal point of mathematical creativity is the ability to solve problems (Silver, 1997). Many attempts have been made to formalize the problem solving process. Most notable among them is Polya, who studied creativity in the 1930's and 40's. His approach to problem solving is at the heart of almost every introductory mathematics textbook on the market today. See for example (Stewart, 2003). Most textbooks use Polya's strategies or strategies based on his work, but do not give him credit for it.

3. Basic ideas and Purpose of the Study

This experiment had several goals. First, it was to increase the student's awareness of the creative process as it occurs in mathematics. Second, to establish a link between creativity and mathematics and also link mathematics to the real world. Third, the students were to evaluate thought processes and creative processes in themselves and others. Fröbel's ideas were introduced and linked to current trends in education, such as manipulatives and the teacher as a guide. Lastly, the connection between the gifts, crystallography and architecture was explored.

The gifts can help students understand that not everything is what it seems to be. Especially, the second gift brings that aspect of geometry to the forefront. The rotation cube that will look like a circular cylinder when spun fast enough gives students at least a brief pause to examine objects more closely. Creativity, although its measurement is difficult, is integral to learning mathematics. I hope that introducing Fröbel's gifts to the students will increase their awareness of the link between mathematics and creativity as well as increase their use of creative (although mathematically correct) ways of looking at geometric facts and theorems.

Mathematics by most people is viewed as a rigid, formulaic subject without any bearing on real life. While it is correct that part of mathematics consists of rules, logical structures and formulas, most of mathematics centers about the ability to develop tools that are applicable to a wide variety of problems. Thus mathematics includes the ability to abstract real world situations, choose the proper mathematical tool for the solution and to interpret abstract results in the light of reality. Most of these abilities are included in Froebel's considerations and teachings.

Creativity in the mathematical process has been studied in young children and early school age children as well as in highly accomplished mathematicians. However, there is a curious lack of concentration on the population between these two extremes. Mathematical creativity in college students has been all but ignored and this experiment is an attempt to close the gap. As far as I know, it is unique in its use of Fröbel's ideas and gifts in a college classroom.

4. Set-up

In a Euclidean geometry class taught at Walsh University in the spring of 2007, 22 students were presented with gifts three through six. The instructions varied with the gifts. Students were asked to form small groups of three to four students. The instructions with gift three were simply to create as many objects of nature, beauty or the environment as possible. Gift four had more specific instructions as some objects such as numbers and letters were excluded. The instructions for gift five were to repeat the process from the last gift with more attention to the process of developing an object. Finally the instructions for gift six told students to each create exactly one object with the gift and the other students had to describe the process of building or creating. Students were also asked to describe their own thought process when creating their object with gift six. After each session students were asked to summarize their experiences with the gift and the instructions. Classroom observations by the teacher were made at the beginning of the class and after each session.

Next, students were required to write a paper including their observations and experiences with Fröbel's gifts, including descriptions of objects and the process of their

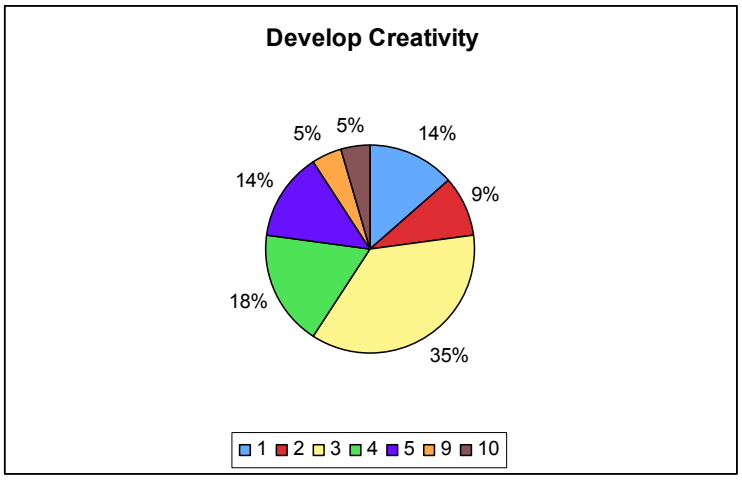
creations. They were asked to provide verbal descriptions as well as perspective drawings of some of their objects. Reading for this paper included a section on Fröbel's life and work. Another part of the paper was a description of the process of creating objects with Fröbel's gifts from observation of others and from the student's own perspective.

Finally, data were collected by asking students to complete a short survey and comments on the class as a whole and specifically the introduction of Fröbel's work. The surveys were anonymous and subjected to standard statistical procedures for small samples.

Since Fröbel's ideas and writings concern mostly kindergarten children and young pupils in elementary school, the concepts have to be adapted to fit into a college classroom. Some of the students noticed the differences in instructions given by Fröbel from the instructions given in class. It should be noted that in college you see your students three times a week for one hour, whereas K-4 teachers will typically see their pupils every day for longer periods of time. Also, the experiment was restricted to a one-semester course.

5. Results

All students rated their experience with Fröbel's gifts and his ideas as very positive and interesting. 91% of the students acknowledged the importance of creativity in learning mathematics, although some students qualified this as being restricted to geometry. Students rated Fröbel's gifts as helpful in developing creativity as a 3.68 on a scale from one to ten with one being the most and ten being the least.



The chart shows the individual answers given by students to the question "How helpful do you think that block play is in developing ones creativity?" 58% of students rated it a three or better.

Students are a little less sure of the importance of creativity in the development of mathematical skills. They rate the importance only as a 4.05. The correlation of 0.56 between the development in creativity and the development of mathematical skills shows that students who recognize Fröbel's gifts as important mostly acknowledge the idea that creativity contributes to mathematical skill. The gifts are seen as helpful in understanding geometry. Although, students

rate the helpfulness of the gifts in this task only as a 4.86, this still shows a positive attitude towards the manipulatives.

Question	Mean	Standard Deviation
1. Did working with Fröbel blocks make this geometry class more interesting? (Y/N)	1.00	0.00
2. Do you think that creativity is important in the study of math? (Y/N)	0.91	0.29
3. Would you consider using Fröbel blocks with your children or with your students? (Y/N)	0.91	0.29
4. How helpful was studying Fröbel to your understanding of Geometry? (1-10)	4.86	1.98
5. Is block play an important activity? (Y/N)	0.95	0.21
6. How helpful do you think that block play is in developing ones creativity? (1-10)	3.68	2.18
7. How helpful do you think that creativity is in developing ones math skills? (1-10)	4.05	2.08
8. What was your comfort level with playing with blocks? (1-10)	2.77	2.11
9. Rate how much you enjoy math. (1-10)	2.55	1.59
10. Rate how much you would enjoy playing with Legos. (1-10)	2.86	2.67
11. As a child did you ever play with wooded block? (Y/N)	0.90	0.29
12. Did this class enhance your understanding of how math is found in the world? (1-10)	4.14	2.26
13. Would more reading material help you in this class? (Y/N)	0.23	0.42
14. How difficult would you rate understanding Fröbel's concept? (1-10)	4.95	2.29
15. Do you feel that you understand Fröbel's method? (1-10)	3.73	1.96

Questions were either Yes-No questions indicated by (Y/N) or on a scale of one to ten indicated by (1-10). The scale was set up so that one was the most and 10 the least, to ensure that students read the instructions correctly. For the Yes-No questions yes was coded as a 1 and no coded as a 0, so that a mean of 0.91 indicates that 91% of the students answered yes to that particular question.

Most of the work submitted lacks in the use of patterns or objects of beauty created with the Fröbel gifts. Students exhibited a strong tendency towards real world objects and neglected the chance to create patterns with the given materials. The work was often centered on thematic groups like animals or football and objects were then created to fit within the chosen category. Even when the creation is passed from one student to another in a group the theme was more than likely to remain constant. I also noticed limited variation from gift to gift. Students tried to recreate the same or similar objects that they created before when presented with a new gift.

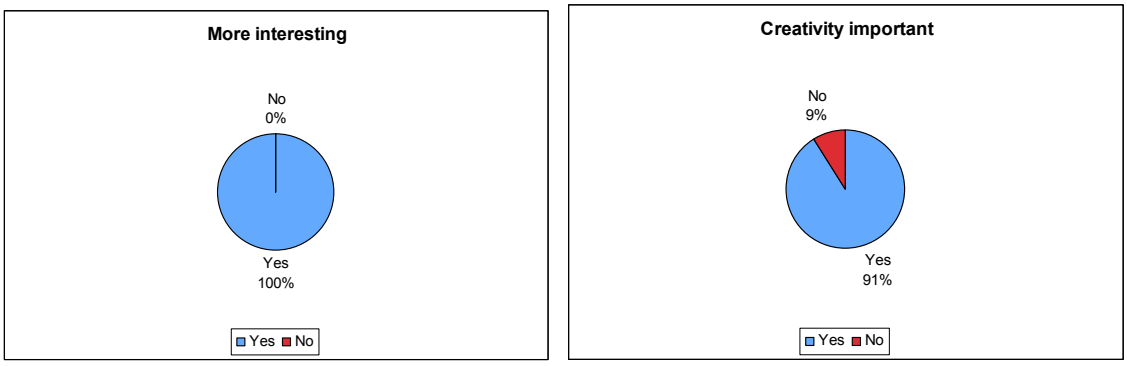
As mentioned above, most students agree with the importance of creativity but do not reflect this in their papers or their work. Similarly, students showed a general lack of abstraction skills. At first they would rather abandon a project than to figure out how to make it work using

abstractions or generalizations about objects. However, as more gifts were introduced, the students became increasingly at ease with the wooden blocks and thus their skills in working with them improved. Some students were able to create very interesting objects with the higher numbered gifts.

At another level, some more general problems were detected. There were difficulties with reading and understanding instructions and a lack of verbal expression skills regarding mathematical ideas and objects. Students have a hard time writing about mathematics and expressing mathematical ideas in written form. They were required to describe the creation of one of their objects without drawing a picture and most students could not give a complete description of the necessary actions and relations of the blocks to allow recreation of their particular object. On the other hand, both class and homework showed an increase in reflective skills and the ability to observe and self-observe.

One student observed that using Fröbel's gifts and its resulting "strategies help develop abstract thinking" which is part of mathematical development since mathematics is the language of abstraction. Part of the problem of developing a good understanding of mathematics is to develop abstraction abilities. On the other hand, students also need to learn to apply abstract mathematics to the real world. Both aspects of the link between mathematics and the real world are important. The students ranked this only as a 4.14.

Understanding of the importance of creativity in mathematics and learning in general went very well. The goal was "to bring out students' creativity and Fröbel's gifts is an excellent way to do that" as one student noted. Further underscored was the importance of creativity by the observation that "creativity allows us to see some of the things we normally would miss in mathematics". It cannot be denied that some students regarded the experiment with a lot of skepticism because "math is all based on logic not creativity".



This chart shows the amount of students judging Fröbel's addition to the geometry class as interesting and the amount of students who judge creativity in mathematics important.

The gifts were praised by students as a tool of understanding geometry and "after working with them it made it easier to understand some aspects" of the class. Also mentioned was the idea that having manipulatives in a college classroom was stimulating to their thinking and raised their interest in the class. Hand-on activities were clearly a surprise for the students in this mathematics class. While most students said that they were comfortable with the gifts, it was

obvious that, especially in the beginning, they had some concern about playing with toys. Overall the students rated their own comfort as a 2.77.

Most students believe that they have a good understanding of Fröbel's ideas and they rated themselves as a 2.97 in terms of understanding. They rate the difficulty of Fröbel's concepts a 4.95. However, the answers to these two questions have a slightly negative correlation coefficient of -0.18, which leads me to believe that few students understand Fröbel's concepts in their entirety.

It was most important for the students to realize that the gifts and the instructions that Fröbel presented together with the gifts were not taken out of thin air but have a firm grounding in the fact that Froebel was a crystallographer who studied nature and its building blocks closely. This gives the experiment with the blocks a new direction and infuses meaning into the seemingly useless limitations and rules that come with the block play.

The awareness of the importance of creativity in mathematics grew noticeably. In the survey 91% of the students agreed that creativity is part of mathematics. Where many students had very little to say about creativity and mathematics at the beginning of the class they admitted to the importance and power of it in the final survey.

6. Conclusions

Overall I think that the students in the class learned many things about creativity and its importance in mathematics. They were exposed to concepts that have been all but forgotten and had a chance to reevaluate some positions they took regarding mathematics. Students have shown a new or renewed appreciation for the mathematical process and the links of mathematics to the real world. Most students regarded the experiment as a success in so far as they were more interested in the class and the material and the gifts actually helped them understand mathematical creativity and geometry better.

It is still not well understood what the triggers for mathematical ability are and how development of mathematical thinking can be furthered, I think that reintroducing Froebel's ideas into the early kindergarten and elementary school curriculum will most definitely help to increase mathematical awareness and creativity. While research into mathematical creativity and creativity in general is taking important steps that will hopefully yield a clear definition and methods of measurement for creativity much is still to be done in that area.

7. Limitations and further research

The class used for this study was a small sample of college students. All students had signed up for a mathematics class and are thus not representative of the whole student body. The study is mainly based on a survey and is limited to observation from the classroom and self-reports in its other data gathering.

In the future I would like to expand the use of the gifts in the classroom to other courses and have access to enough of the gifts so that each student can have their own set for experimentation. I would like to use tools that more accurately measure mathematical creativity. The next experiment will contain a pre- and post-test about attitudes towards mathematics and creativity and about the perception of a connection of disconnect between the two.

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[2] Editorial Note: ZDM just released a double issue on interdisciplinarity which contains a section focused on creativity. The interested reader can follow this link
http://www.springerlink.com/content/g71m25052028/?p=eb8fe5ea1dae4145af89a82f6bbd5f3b&pi=0