Reflections upon Teaching a Poorly-Conceived Lesson

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Abstract
Using a "failed" mathematics lesson as a mini-case study, I explore some of the challenges inherent in teaching an inquiry-style mathematics lesson. The ensuing discussion centers around two issues: the preparation and subsequent scaffolding involved in guiding students to desired understandings, along with the tension inherent in "telling" and "not telling" in an inquiry-style pedagogy. I resort to personal reflection, as well as to Wenger's (1998) theoretical construct of *participation* and *reification* in revealing the underlying link between the manner of lesson preparation and consequent engagement of students during the enactment of the lesson.

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One of the difficulties in attempting to transform one's teaching practices from a traditional lecture format to a more student-centered mode of instruction is that one oftentimes relinquishes a certain degree of control and predictability in the classroom. As opposed to simply planning out a lecture with the expectation that students will ask questions when they do not follow *the course of reasoning as set out by the instructor*, the new practice requires one to more carefully map out—that is, to anticipate and be prepared for— various contingencies: in particular, such issues as, in what manner might the ideas be best embedded within activities (as opposed to directly telling them), what degree of scaffolding might be necessary and optimal for the different levels of students in the class, in what manner might one want to go down divergent learning paths depending upon student input, and so on. As such, in opening up the classroom discourse, by its very nature, student-centered teaching practices introduce new and necessary skills for the teacher to acquire both in preparation and in interaction with the students (see Lampert, 2001).

When the lesson plan has been carefully thought out in these terms and more, the teaching itself can be a real thrill. It affords a richness of dialogue, and even learning opportunities for the teacher that lecture-based teaching does not often enable. On the other hand, when done poorly, this other way of teaching can leave both teacher and students alike in pedagogical predicaments that the traditional approach safely steers clear of for the most part. I am particularly interested in this latter case, of when things are not aright, of when such *attempts* can go messily wrong. I am interested in this, for I believe that it is in exploring *when* and *how* a particular pedagogical approach fails that one sometimes comes to a deeper understanding of how it might come to succeed.

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In this short paper, I would like to relate my own teaching experiences using an "activity-based" mathematics lesson that in my opinion came up short. The lesson is chosen not for its uniqueness, but for how it epitomizes a collection of "failed" lessons from a semester's teaching. In describing my interactions with the lesson, I hope to begin characterization of a few choice principles in lesson planning that may perhaps carry over into curriculum design.

A Post-Introductory Prelude

Prior to my involvement in a collaborative lesson-planning group at a university-level mathematics education department, I taught for approximately six years at the collegiate level within mathematics departments. The primary distinction between my prior teaching experiences and my currents ones are as follows: whereas I planned and taught alone before, I am currently involved with three to four other instructors in the lesson planning.[1] In addition, our lessons are more of the activity-based lessons, while I used to teach mostly in a traditional lecture or recitation-style format.

I want to relate here a series of incidents from my first semester during this transition that I think relevant to the discussion to follow. First, to set the scene: the teaching concerns a sequence of mathematics content and methods courses for prospective elementary teachers. As mentioned, part of the teaching involves collaborative planning of the lessons. We gather together, on average, once a week for one to two hours to discuss our learning goals and the manner in which we would like to structure our lesson to accommodate those learning goals.

I recall one particular lesson, sometime during mid-semester that I somehow was not able to "bring to life." I found myself ill-prepared for fostering the kind of lively discourse among the students that up until that point, I felt I had successfully encouraged. I remember having two competing thoughts: the first was that student-centered pedagogy was hard, and that perhaps it was my inexperience with this type of teaching that left me *feeling* somewhat a failure as a teacher. The second thought was that maybe the lesson didn't really fit my *pedagogical style,* and it wasn't that I was a failure as I thought.

The real revelation occurred during our next planning meeting. Somehow, I let it be known that my confidence as a teacher was waning. Then one of the other instructors confided that she too felt the same way, and immediately, yet another instructor admitted to similar sentiments. The fourth instructor, who had been involved in the group planning process longer than two out of the three of us, mentioned that *the lesson plan had been problematic for him!*

What struck me about the last colleague's comment, and also the convergence of our so-called failures in teaching, was that until that point, I had been conceiving teaching in terms of individual and highly idiosyncratic teaching abilities, or more specifically, in terms of one's *personal ability* to bring a lesson to life in the classroom. Somehow, in the previous lessons when the lesson seemed to go well, I had secretly thought, *what a great and talented teacher I am that I am able to bring these half-baked lessons to life and to engage the students in lively and meaningful discourse!* And suddenly, I realized that there could be a lot more to it than that. It dawned on me that my "performance" abilities might have had a little less to do with some innate talents, and that instead it was more the lesson plans that seemed either to offer the space within which to bring out my capacities as a teacher or constrain them by their ill-planned nature.

As if that one experience were not enough, during subsequent lessons, whether I felt, again, the same old sense of success or of failure, I noticed that my experiences were regularly matched by the experiences of my colleagues. I found myself fascinated and intrigued by this fact, that even with our wildly divergent teaching styles and teaching backgrounds, we were having remarkably similar teaching experiences over each of the lessons. What this did for my thinking was that whereas I had previously placed much of the credit as well as blame of teaching upon my performing abilities (and in some ways upon my personality), I began to think instead of the lesson plan as the more meaningful point of focus in terms of what worked and did not work in a lesson.[2]

In retrospect, I think that the lessons seemingly failed when we had not carefully enough thought about the relevant mathematical and pedagogical issues involved and had instead glossed over some crucial details in our planning. It is not a particularly deep or earth-shattering insight, but I do think that it is an important one. In fact, it is the primary point that I now hope to elaborate upon more concretely in the context of a so-called failed lesson. I hope also to make a secondary point, again in the context of the lesson, regarding the tension between "telling" and "not telling" in mathematics teaching. I will first present the lesson, then an account of my own experience of teaching it, followed by reflections upon the lesson.

The Lesson[3]

The lesson motivates an alternate algorithm—sometimes referred to as the *common denominator approach*— to the invert-and-multiply procedure for division of fractions. Here is an example:

$$\frac{2}{3} \div \frac{1}{4} = \frac{8}{12} \div \frac{3}{12} = 8 \div 3 = \frac{8}{3}.$$

The idea is that when given a division-of-fractions problem, one first finds the common denominator. Once both fractions are rewritten with a common denominator, only the numerators need be considered in the division—that is, the denominators can essentially be "ignored" in the subsequent calculation.

To begin, the exploration lays out its intent by stating, "The purpose of this exploration is to help you to understand this [alternate] algorithm and why it works" (p. 154). Its aim is for students to arrive at such an understanding through a process of *discovery*, as will be made clear through the ensuing description. The first task begins by offering five story problems, all of which model a division operation. Here is a modified example:

> *Rick has 4 lbs of sugar. Each time he wants to make a cake, he uses 2/3 lbs of sugar. How many cakes can he make with the sugar he has?*

For each of the story problems, the students are asked to represent the problem and its solution using a diagram. One could imagine a student beginning by drawing four rectangles to stand for the four pounds, where each of the rectangles is then partitioned into thirds to accommodate the eventual "taking away" of the 2/3 pounds at a time. After counting that this can be done six times, the student would conclude that the answer (by use of a measurement model for division) is six.

The activity then asks the students to "consider carefully" and "describe" what "[they] did in order to arrive at [their] answer." The students are subsequently asked to "try to connect what

[they] did on paper to how the problem could be solved using only numbers." One hint is offered in "For example, in the first problem, the original number sentence is 4÷2/3, but regardless of the diagram you draw, you will divide each of your four units into thirds, and thus you are now solving 12/3÷2/3" (p. 154)

An ideal description in response to the prompt for the sample problem above would sound like this (I have italicized the key turn in reasoning):

> I drew four boxes to represent the four pounds. I needed to take away 2/3 pounds at a time, and see how many times I could do that. So, I needed to partition each of the boxes into thirds. So, instead of just 4, I now had 12/3. So, the problem really is 12/3÷2/3. At this point, I am asking how many times can I take away 2/3 from 12/3. *But at this point, it doesn't matter that each of the pieces are called thirds. It's really just a matter of taking away two pieces at a time from 12 pieces, and seeing how many times you can do that. Whether you call those pieces 'thirds' or 'ones'—or even 'fourths' or 'fifths,' for that matter—it's still the question of "how many times can you take away 2 from 12?" They would all give you the same answer of 6. So, 12/3÷2/3 is the same as 12÷2."*

Lastly, the activity closes with the following instructions: "After completing the five problems, look for commonalities in all the problems that lead to a generalization (rule) that you could use in all the division problems" (p. 154). The goal, obviously, is for the students to come to know and to understand, through a process of discovery, the 'common-denominator algorithm.'

How the Lesson Actually Proceeded

Here is how the flow of this lesson transpired from my perspective as the instructor. First, I noticed how most all of the students were able to represent, with relative ease, the problem and its solution through a diagram. We had already covered the topic of representing division solution strategies with diagrams in previous class meetings.

But when asked to describe what they did in order to arrive at their answer, suddenly no one in the entire class appeared to know what the problem was "getting at." Many of those who vocalized their confusion were able to describe what they had done in their diagrams, but mostly in procedural ways, such as, "I drew 4, and then I took away 2/3 pieces six times, and so the answer is six." When pressed, some students were able to mention the repartitioning of 4 into 12/3 but little more. Not one student was able to describe the key turn in reasoning that this activity was meaning to raise. We had established a class norm of justifying mathematical claims by this time in the semester, but at the same time, previous discussions had centered on much simpler and more straightforward ideas and conjectures.

I recall thinking to myself that the lesson was not working. It was not *leading* the students to the desired mathematical insight and thinking as we, the instructors, had expected. I tried asking various questions that I thought might lead to the desired key turns in reasoning, but to no avail. Unable to generate the appropriate kind of scaffolding question that might gently *guide* the students' thinking toward the desired end, I ended up *telling* the students what the activity was getting at (the common denominator approach) and showed how the diagram for the first story problem 'proved' that such an approach worked. I was essentially modeling for the students what they were to do with the remaining four problems. During my elaboration, I guessed that less than half the students had grasped the argument I was expressing, though at the time, I was at a loss as to how I could deal with the situation in a better way. Secretly and with some distress,

I was hoping that those students who understood my explanation would later explain it to those who didn't during the subsequent group work.
After allowing for some time for group work on the remaining problems, I asked a few students to explain their explanations in regards to problems two through five. As expected, few if any of the students showed that they had truly grappled with the reasoning process and instead had somehow *proceduralized* the reasoning into something fairly routine and superficial. They now knew *how* the new common-denominator algorithm itself worked and could be applied, but the conceptual reasoning as grounded in diagrammatic referents—more or less, the *why*—somehow got lost in the process. And again, my attempts at having students explain the *why* were met with confused and somewhat hostile glares. Interestingly, all four instructors had similar experiences with the lesson.

Reflecting Upon the Lesson
The central question for me is this: what might be the lessons learned from such a lesson? What general principles might I extract from this experience? In reflecting back, I think of the activity as assuming too much of the students' ability to see what the designers of the activity clearly saw. While we, the instructors, knew *how to look* at the reasoning process involved in the use of the diagram in order to see, or extract out, a justification for the alternate algorithm embedded within it, the students, in fact, had no idea of what to even look for. Nowhere in the lesson, up until that point, was it mentioned that the point was to derive and justify the common-denominator approach to division of fractions. In some ways, it could be asserted that the pre-service teachers were asked to stare at their diagrams and somehow chance upon a half-baked proof for an algorithm whose existence they had no knowledge, nor awareness of—what, in retrospect, I might suggest as a tricky and next-to-impossible task.

A question arises: if indeed I have captured some essential aspect of the lesson—specifically in how such an activity purposefully withholds important information that might otherwise help a student to "see" the mathematical relationship or concept—how could such an activity ever see the light of day, especially in a published textbook? Of course, a group of us chose this activity for our own lesson! So, though I might not capably answer the question in regards to its publication, I might at least speculate as to how a group of us might have overlooked its particular shortcomings. I do so through a particular theoretical framework.

In his book *Communities of Practice (1998)*, Etienne Wenger discusses the interplay between *participation* and *reification*. Participation is defined as the "complex process that combines doing, talking, thinking, feeling, and belonging. It involves our whole person including our bodies, minds, emotions, and social relations" (p. 56). Meanwhile, reification is defined as the "process of giving form to our experience by producing objects that congeal this experience into thingness" (Wenger, 1998, p. 58), and would include mental objects such as concepts and even words. As an example, a book is a reification of someone's thinking, with the act of thinking being a form of participation. Even a person's understanding of something is a kind of reification, as long as it has *solidified* into something that one might call an understanding, whether correct or not. Wenger points out the interplay between the two, that participation often leads to a reification, which in turn affords further participation, and so on. So a lesson plan can be looked upon as one very concrete reification of the act of participating in discussion and/or preparation, while that same lesson plan also gives rise to the kinds of participation (in the form of thinking, discussing, solving, and so on) that are available to students. Wenger also points out

the notion of *premature reification*—that is, the idea of arriving at reifications before sufficient participation has been realized.
Thus, in terms that Wenger has introduced, one way to describe what happened during the lesson is to say that the students had prematurely moved to reifying their understanding before sufficient participation with the ideas had occurred. Telling a student to "carefully consider" what they had done in making a set of diagrams is not a sufficient prompt toward the kind of full participation necessarily for the kind understanding (reification) sought of the reasoning involved. Without sufficient participation, the resulting understanding is oftentimes fragile, as Wenger posits. Or in the case of the lesson, it was not in any way complete nor deep.

Put differently, one could say that *the activity's intent of having the students come to an understanding of why the algorithm works was circumvented due to insufficient engagement with the key ideas.* Of course, expecting students to "discover" a new piece of mathematics without sufficient scaffolding, or support, more often than not will fail to foster the desired engagement.

Yet, one might also go backwards in this back-and-forth chain between participation and reification. That is, *insufficient grappling with the mathematical ideas amongst ourselves (the instructors) during our planning sessions appears the culprit in the deficiency within the lesson (the reified object)—a deficiency which led to our students mirroring our own insufficient engagement with the mathematics.* That is, to say that the lesson plan "failed" to engage the students in *full participation* (leading to a failure to reify a desired understanding) is in some ways to say that the participatory act on the part of the lesson planners was not deep or thorough enough. One mirrors the other.

Another relevant factor, I believe, was the rather unreflective grappling with the role of telling and not telling in mathematics pedagogy. It occurs to me that it would be fairly easy to internalize the message of reform as, "Let students discover the mathematics, rather than telling them." This approach to mathematics teaching, no doubt works in some contexts, but it also fails quite miserably, as can be seen, in other contexts such as this. The challenge is in knowing when telling or guiding would unnecessarily clamp down on what otherwise might be productive thinking and when it would be beneficial, perhaps even necessary, and lead to fruitful learning. It is certainly not a trivial issue (see Chazan & Ball, 1999 for a discussion on the use of "judicious telling").

In Summary
Two critical points emerge. The first and primary point relates to a level of detail in support offered or not offered within an activity—sometimes, referred to as "scaffolding." When viewed from Wenger's framework (reification and participation), implicit in a presence or lack of scaffolding within a lesson is the presence or lack of careful thought (or in the discussion, if group planning) around the planning and construction of the lesson. Another way to think of it would be to say that *the lack of engagement on the students' part during the lessons makes manifest the shortcomings in the lesson plans, which in turn reflects the deficiency in the engagement with the relevant mathematical ideas during the designing of the lesson plans themselves.*

As a teacher, I have in the past thought, *if it takes me, the instructor, a little effort to solve a problem, then chances are half the students won't be able to solve it.* Perhaps others have had similar (and useless) thoughts. In light of the point I am raising, I might offer an amendment to the thought and offer it instead as a workable pedagogical principle: *if it takes a teacher a little effort to solve a problem (solved, likely through tacit understanding of the underlying concepts), that's likely an indication that s/he needs to be very* *clear and explicit on the mathematics at a finer-grained level, appropriate to his/her students' mathematical maturity and understanding. That is, there must further engagement (participation) with the pertinent mathematics. Further, such a teacher also needs then to take his or her finer-grained understanding to reconstruct a scheme for approaching the problem with an eye on how best to elucidate, or bring about understanding of the key points.* Through careful thinking of these issues, the teacher might be better prepared with appropriate scaffolding questions; or for the curriculum developer, these key insights might be contained and explicitly surfaced within the activity/lesson, if the implementing teacher is to have a chance at successfully bringing the lesson to life.

The second point relates to the pedagogical awkwardness that results in withholding a piece of information that would otherwise help the students to "see" a particular mathematical concept or relationship, especially under the guise of allowing for "discovery" or "not telling." The essential factor, in this case, might not be in the *form* of the teaching – telling or not telling—but in whether telling, or not telling, is supporting or hampering the engagement level of the students. I would posit that it is the degree of student engagement, and not the *form* of the teaching, that stands as the first principle in both lesson/curriculum design as well as in teaching. It is the question of how best to raise and maintain a high level of cognitive engagement, and in turn reflection upon subject matter that tells us what pedagogical actions a teacher might adopt in a teaching situation.

In closing, I would like to note how I began by describing this and other such lessons as "failed" lessons. A point worth making is that a so-called "failed" lesson, in fact, has the potential to becoming a meaningful *success* toward professional growth if one takes the time to extract out new learnings from them, and perhaps that is one underlying message of this entire tale.

References

Bassarear, T. (2001) *Mathematics for elementary school teachers: Explorations (2nd ed.),* Boston: Houghton Mifflin Company.

Chazan, D., & Ball, D. (1999). Beyond Being Told Not to Tell. *For the Learning of Mathematics, 19*(2), 2 - 10.

Lampert, M. (2001). *Teaching problems and the problems of teaching.* New Haven, CT: Yale University Press.

Stigler, J., & Hiebert, J. (1999). *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom.* New York: Free Press.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* Cambridge: Cambridge University Press.

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Endnotes:

1. I am not speaking here of "lesson study," as in Japanese lesson study (see Stigler & Hiebert, 1999), but instead, I mean planning each of a semester's lessons together with colleagues, and all of us teaching that same lesson.
2. An interesting benefit that I have noticed regarding such a view of teaching is that one feels less threatened by observers and criticism. Whereas before this experience I might have felt that any criticism of the teaching was directed at me as teacher, I have begun thinking of criticism as being pointed more to the lesson, and less to myself. This in turn has helped ease the transition from working in isolation as a teacher to becoming part of a larger teaching community.
3. The lesson consists essentially of going over a handout ("Exploration 5.12: An alternative algorithm for dividing fractions) from Bassarear (2001). As such, I will refer to it interchangeably as "lesson," "activity," or "exploration."