

Non-linear functions in secondary school of lower qualification level (German Hauptschule)

Astrid Beckmann¹

University of Education, Schwäbisch Gmünd, Germany

Abstract

This article reports of the effectiveness of introducing non-linear functions in a German Hauptschule. Some research based recommendations are provided for practitioners with the goal of improving student competencies in lieu of PISA..

Keywords: classroom experiments; non-linear functions; non-linear relations; student competencies; PISA competencies; German Hauptschule

1. The Situation in the German Hauptschule (secondary school of lower qualification level)

The results of international comparison studies like PISA recommended that German educators increased their focus on the mathematical competencies of students in secondary schools of lower qualification level (Hauptschule). According to the results of PISA 2003 not even one-fifth of the students achieved level 1 of competence. Tasks at level 1 “demand to gather information from a simple table given in standard form or from a simple graph and to execute simple calculations which refer to relations between two familiar variables” (PISA, 2004, p 56). A further striking break is to be seen in the field of level 3 to 4. While almost a quarter of junior high school students (German Realschule) are at level 4, this does not even apply to 5 % of the students of the Hauptschule - apart from higher levels which are hardly reached by one. Tasks of level 4 require additionally “to argue also in less familiar functional contexts and to

¹ Astrid Beckmann

Professor Dr. habil.
Department of Mathematics and Computer Science/ head
University of Education Schwäbisch Gmünd
Oberbettringer Straße 200
D - 73525 Schwäbisch Gmünd
GERMANY
Tel.: +497171983359
email: astrid.beckmann@ph-gmuend.de
Web: <http://mathematik.ph-gmuend.de/lehrende/beckmann/>

communicate these arguments as well as to deal with given linear models of real situations” (PISA, 2004, p 56). Figure 1 shows a linear task, (not necessarily) a familiar real situation.

The picture shows the movement of a car.
In which traffic situation is the car?

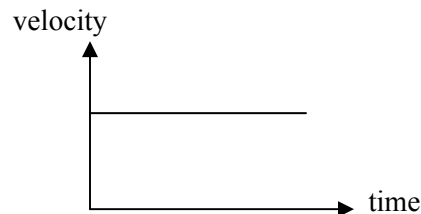


Figure 1

A characteristic answer from our own random sampling at the Hauptschule is the following: “The car is always driving straight on”. This answer is in accordance with the observations from PISA and further studies according to which pupils interpret rather the optical picture than the functional interrelation (compare known racecourse task in PISA). Since at the same time they describe the graph in figure 2 as follows: “The car is rolling downhill.”

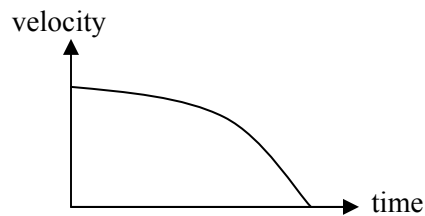


Figure 2

The question is how can Hauptschule students be supported in their learning in order to reach level 4 easier and be more successful?

2. About the importance of non-linear functions

Thesis:

For the achievement of level 4 the acquaintance with non-linear functions is beneficial.

A good reason for this thesis is that so far in the Hauptschule only linear functions are treated and take up a big space of time in the curriculum in comparison to other function types. These students experience functional contexts as mainly linear. This may lead to a restricted view and functional thinking becomes dispensable. For example the overemphasis of linear functions may lead to the perception that any functional coherence can be described by a straight line. Experiences from school projects show that students often think accordingly. If you ask them to demonstrate the relation between air volume and pressure graphically they mark a straight line with positive gradient even though the relation is inversely proportional. Furthermore the presumption can be supported that even two points are enough to specify the graph of a function. This has also been noticed in school projects when cubic relations have been rashly interpreted

as straight lines. Examples like these show that working with different types of functions requires more flexible thinking. It is not clear from the beginning how the graph is running, which output belongs to a certain x etc. Non-linear functions require thinking in changes and connections and lead to aspects of covariation (DeMorois & Tall 1996; Malle, 2000).

However non-linear functions can be complex and have typically been described algebraically. For example, intensity of lighting diminishes quadratically with the distance from light source. In the Hauptschule the functional term should not simply be the objective but the functional correlations with their dependencies, changes etc should be emphasized. However there is a certain difficulty included which is related to the graphical description. A better access to the functional connection can be produced by equalizing curves plotted instead of points connecting lines. According to our experience it has to be reflected before and during class in order to approach this result.

3 .The School Project

In November 2006 a first testing of the treatment of non-linear functions took place in a Hauptschule in Baden-Württemberg (Ostalbkreis, Germany). In different stations 9th grade students learned about non-linear functional relations in experimental activities. The students had already been used to experiments. At the end of 8th grade there had already been a course about linear functions only. Experiments had been chosen because of prior experiences in this area conducted in different school types with positive results from such experiments in connection with functions (Beckmann 2006, 2007). In the performance of these experiments the aspects of the contents of function like correspondence and covariation can be experienced by action. For example, in dipping a ball with a certain radius into a jug filled with water a certain water volume is edged out. The experience is made that any radius corresponds to a certain volume. With a running car the concurrent change of line and time, i.e. covariation, can be witnessed directly etc. (see Dubinsky& Harel, 1992; Vollrath 1978). In the school project the experiments were conducted together with worksheets. Worksheets proved to be very helpful in order to activate certain trains of thoughts, which could be deepened in the final presentation. Main aspects are summarized in table 1.

Aspects	Example
Everyday activities Forming the Thesis	Pumping a closed bicycle pump and then discuss on it
General Task	Describe the correlation between air volume and air pressure. Check: does the correlation confirm your answer to the above question? Describe the special characteristics of the correlation.
Operation of the Experiment	Get familiar with the parts of the experiment and the components of the measuring possibilities.
Correspondence	Adjust a certain value for the air volume and record the value in the box. Read the corresponding pressure. Record ...

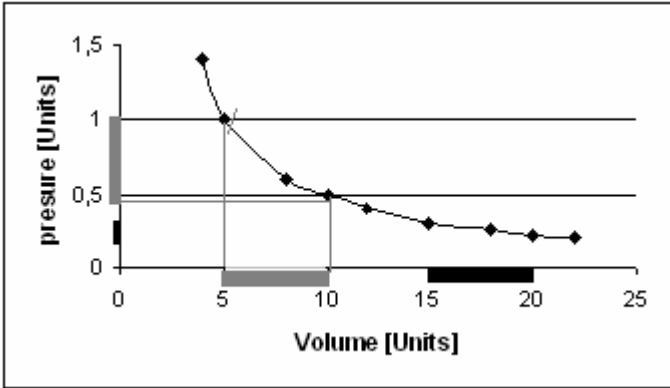
<p>Change between different forms of presentation</p>	<p>Act in a systematic manner and measure. Record in the table.... Characteristics of the table... Enter the values from the table in the system of co-ordinates. Regard the graph. Describe it.</p>
<p>Correspondence</p>	<p>Which pressure corresponds to a volume of 5 units?</p>
<p>Covariation</p>	<p>Highlight in the graph by a thick line only the change from 5 to 10 (from 15 to 20) volume units at the x-axis and only the corresponding change at the y-axis. Compare the particular changes...</p> 
<p>Closure</p>	<p>Document the results of this station clearly and neatly on the prepared poster. Remind the general task.</p>

Table 1 Structure of a worksheet

In the school project the following experiments were chosen out of a big collection (Beckmann 2006)²:

- Functional relation between air pressure and air volume
- Relation between intensity of light and distance of the light source
- Relation between radius and volume of cylinders of the same height
- Relation between height and time of a falling ball
- Relation between distance and time of an accelerated car
- Relation between force and lever of force

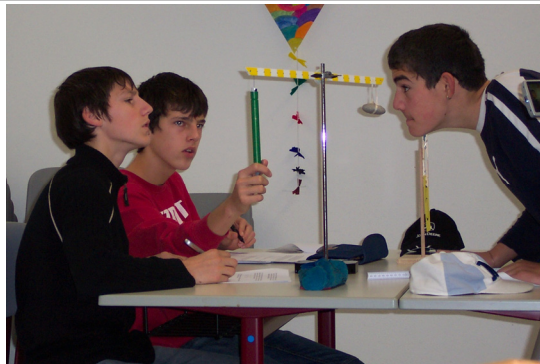


Figure 3
Experimenting Students of the Hauptschule

Implementing these experiments needed two double-lessons, in which the students experienced two or three relations. The division into student groups was decided by the teacher. Before starting the experiments a short introduction including questions of graphical description was given.

4. Observations and results

The lessons were video and audio taped, the worksheets were evaluated and single episodes were recorded. The evaluation shows that the experimental activities in non-linear contexts led to some aspects according functional thinking. The difference to the (expected) simple proportional relation and the missing possibility of predicting more values led to discussions. The students experienced and applied the idea of covariation. They recognized that only two values do not lead to an indisputable statement about the function.

Reflection about the table; extracts from worksheets:

“The lower the distance [height of fall], the shorter the needed time [of the falling ball].”

“The distance does not double.” [comparing the distances of the accelerated car between 1 and 3 s with the distances between 2 and 4 s time of going].

Forming a thesis using the aspect of covariation:

S1: Now we hang it at the end.

S1 measures 25.5 cm

S2 measures: 0.6 N. Actual this has to be at 36 cm.

S3: It should be one less the more you hang it at the back.

S2: Now it is 18.

S1 measures: It is 1.3 N.

S3: The next should be 0.1 N more.

S1: At 13 cm it is 2.1 N.

Checking the thesis and gaining in through experiments; extract from a taking down:

² As agreed with the mathematics teacher Mr. Arthur Litz, Sebastian-von-Drey-Schule in Ellwangen-Röhlingen

The students discuss the relation between the intensity of light and the distance from the light source. In a first measurement they have obtained two values.

S1: Normally it is not proportional

S2: Eh, than let's do it; then we will know.

S1 draws a system of co-ordinates. Together the students discuss the division of the axis and plot it. Finally the measurement is filled in.

S2: That is not proportional.

S1: Sure?

S3: Except that the line runs like this.

S2: That is not proportional, as I said before.

The students take more tubes and take many measurements, before they bear out their assumption.



Figure 4

Measuring the intensity of light at a tube

The following final report of a student shows that he tackled with correspondence and covariation:

“The second project - we called 'light and tunnel'. The more the car drives into the tunnel, the more it becomes dark. The first tube had a length of 9.7 cm. When we held it at the window the intensity of light was very good 36 lux. When we held a tube with 30 cm length at the window, we could only measure 0.1 lux. In an extra drawn graph we could read exactly, how much intensity there is in the beginning of the tunnel and in how far it decreases.”

5. Summary and Perspectives

Starting point is the thesis that working with non-linear functions supports flexible functional thinking, thinking in correspondences, dependencies and changes. This kind of thinking is seen to be a condition for competencies on level 4, which only few students of the Hauptschule reach. An access to non-linear functions is given by simple experiments. While doing experiments the students can experience the aspects of functional relations.

In this school project 9th grade students of a secondary school of lower qualification level (German Hauptschule) were confronted with non-linear relations in experiments. Obviously the non-linear functions led to discussions about dependencies and the course of the graph. Possibly the experiences of the first run of the project played a role, in which the students learned linear

relations only. Summing up the worksheets with detailed questions and the final classroom presentations with critical discussions were an important basis for a fruitful tackle with functional relations. A certain difficulty was the inclination of students to draw a straight line instead of equalizing curves. This could be cleared up in appropriate follow up discussions.

This single school project cannot lead to a conclusive answer to the question to what extent the non-linear experiments led to success regarding competence level 4. But the results show that non-linear functions are an appropriate theme for 9th grade students in a secondary school of lower qualification level. It gives great hope.

References

- Beckmann, A. (2006): *Experimente zum Funktionsbegriffserwerb im Mathematikunterricht der Sekundarstufe I und Anfang der Sekundarstufe II*, in press.
- Beckmann, A. (2007). *Funktionsbegriffserwerb durch Experimente*. In press in *Mathematik Lehren*.
- DeMorais, PH.& Tall, D. (1996). Facets and Layers of the Function Concept. Puig, L., Gutierrez, A. (Eds.). *Proceedings of the 20th Conference of the International Group for Psychology of Mathematics Education (PME 20)*, 2, 297-304
- Dubinsky, E. & Harel, G. (Ed.). *The Concept of function. Aspects of epistemology and pedagogy*. Mathematical. Association of America.
- Malle, G. (2000). *Zwei Aspekte von Funktionen: Zuordnung und Kovariation*. *Mathematik Lehren*, 103, 8-11
- PISA-Konsortium (Hg.): *PISA 2003, Der Bildungsstand der Jugendlichen in Deutschland – Ergebnisse des zweiten internationalen Vergleichs*, Münster (Waxmann)
- Vollrath, H.-J. (1978). Schülerversuche zum Funktionsbegriff. *Der Mathematikunterricht* 4, 90-101