

Understanding Polygons and Polyhedrons Using Flexagons

Aaron Tekulve

Introduction

The goal of this paper is to help students understand simple polygons and simple polyhedrons. First the project within this paper involves having students look at polygons. Though much of this information should have been learned in the fourth grade it is still important to review this material. Having the students define certain shapes illustrates their true understanding of the subject. The second part to the project within this paper is to use the student's knowledge of polygons and build polyhedrons. In this paper the students will only have to concern themselves with squares and equilateral triangles. Here they will have to build constructions known as "flexagons." A flexagon in short is a polyhedron constructed by using tag or poster board and cutting out patterns with wings and rubber banding them together. The third part to the project within this paper is to take all of the polygons and flexagons/polyhedrons and make them into a mobile. The mobile works as a nice way to end the project by displaying the knowledge these students have attained in a creative manner.

The Geometry Standards for Grades 3-5 from the NCTM Principles and Standards for School Mathematics web site states; students in grades three through five should be expected to know how to identify, compare, and analyze attributes of two- and three-dimensional shapes and develop a vocabulary to describe the attributes of these figures. Students should also be able to classify these assorted shapes in accordance to their properties and build a distinction between the various classes of shapes. Not only should students understand the vast attributes of two- and three-dimensional shapes they should also be able to create them. Students should be able to create these shapes by drawing them as well as building them. Also students should understand that three-dimensional objects are constructed from two-dimensional shapes.

Montana Mathematical Standards for Elementary Students

Mathematical Standard:

- 3.1.5 Identify and classify triangles, quadrilaterals, pentagons, hexagons, and octagons according to their attributes.
- 3.1.10 Identify and describe solid figures-cubes, prisms, pyramids, cylinders, cones, and spheres.
- 4.2.1 Determine the perimeter, area, and volume of shapes and solids by counting segments, square units, or cubic units.

Mathematical Objectives:

- To physically investigate and analyze the attributes of geometric 3-dimensional figures.
- Students will recognize and be able to classify various polygons and polyhedrons.
- Students will understand how to measure perimeter, area, and volume of shapes.

Both the NCTM and the Montana state math standards and principles run coherently with each other. Though the Montana state (school district 6) math standards and principles are not as detailed as the NCTM's standards and principles they both cover the same key points. The concepts in this paper run parallel with the standards given by these two sources for educating third through fifth graders in geometry.

I believe the project within this paper demonstrates a logical and creative way of learning polygons and polyhedrons. Not only does this project teach the students about polygons and polyhedrons but it gives them a hands-on-approach with the subject. This project serves as a visual, verbal, and a kinesthetic form of learning while including and allowing multiple intelligence learning styles. Not only can this project apply to students of various learning styles it also allows the student to be creative and use their imaginations making math more appealing and fun.

Before I discuss the various sources from where I gathered information I would like to preface it by saying that one of the key factors to the project discussed in this paper is the idea of it working with visual, verbal, and a kinesthetic forms of learning while allowing multiple intelligence learning styles. Since this aspect is highly valued in this paper I have chosen certain pieces of literature discussing learning styles in the classroom.

Literature Review

Being able to understand and work with various learning styles students have in a classroom is a very valuable tool. *Teaching Math Effectively to Elementary Students* is a journal primarily discussing various learning types. It discusses the stress many students have in the classroom setting. Many elementary students feel ashamed, guilty, and depressed when they are not able to grasp a concept conveyed to them by a teacher. The problem usually lies in the method the teacher is using to teach the subject. This article addresses the basic forms of learning, which are visual, verbal, and kinesthetic. Once illustrated how frustrations in the classroom can occur this journal discusses an assortment of ways to integrate these different learning styles in the classroom. I found this journal to be a very valid source to gather information from because of its discussion on different learning styles. One of the greatest aspects to the project in this paper is its ability to incorporate each of the learning styles into one session of learning. A very productive style of teaching is to have the students physically create the objects they are studying, though it is important to first give them a strong vocabulary and understanding of what makes up the shapes they are creating.

Oberdorf & Cox (1999) presented many valuable pieces of information on the importance of teaching geometry. One of the main themes discussed in this article was the idea of misconceptions in geometry made by elementary students. Some of the reasons cited dealt with limited experience, false information from parents, and the lack of exposure to the proper vocabulary. This article then explains ways to properly teach geometrical concepts to elementary kids. The idea given that I found most useful dealt with square, rectangles, trapezoids, triangles, and three-dimensional objects. Given this project works primarily with polygons and polyhedrons, the information in this journal became very effective. Many of the ideas on how to identify these shapes dealt with constructing them or finding them in the classroom. This allows the students to internalize what they are being taught and see how it links to the real world. "Geometry helps us represent and describe in an orderly manner the world in which we live" (NCTM, 1989, 48). In the end the goal is to have a student look a cereal box and

say this is a rectangular prism made up of quadrilaterals, or to be more specific rectangles. This connection to the physical world is very important because students are able to see the rationale behind studying this subject.

Battista (2002) shared a great amount of information dealing with teaching geometry using computer software to ways of educating elementary students in geometry and having them internalize what they are being taught. In this article the van Hiele theory is discussed quite thoroughly. The van Hiele theory deals with a students moving through several qualitatively different levels of geometric thinking. These levels consist of recognizing shapes, understanding properties of shapes, being able to classify shapes with their various definitions, and being able to prove certain shapes by using proofs. This article also deals with the idea of kinesthetic learning. One example deals with a hinged-rod parallelogram. By moving the hinged-rod parallelogram around a student is able to see the various equilateral parallelograms that can be made. Also there is much discussion in this article about the use of technology in a classroom using a program similar to Geometer Sketchpad. This article gave a detailed discussion on how students can visual and create various objects. Being able to create these objects with this computer program also gives the students a chance to see the different possibilities of certain shapes such as trapezoids, parallelograms, rhombuses, etc. All of this is very powerful when teaching geometry because of the direct connection students can play in their own learning process. Plus students are given the chance to interact with geometry on a new level making geometry more appealing while still educating the students. This article served as a great tool because of the many ideas focusing on the teaching of geometry. Various ways of teaching geometrical concepts is useful to a teacher as well, for it allows them one more way to educate their students without having to rely purely on lecture.

Koester (2003) described many interesting ways of teaching third to fifth graders about geometric figures, primarily polyhedrons. The Van Hiele Model of Geometric Thinking is used in this article as well to stress the development of geometric thinking is more dependent on instruction than on age. This article thoroughly focuses on using various activities to teach geometry and have the students understand geometry. One of the activities used in this article dealt with the connection of straws. One of the major concepts behind the activity is the idea these polyhedrons are being constructed from polygons. This article was quite detailed in its overall explanations of how polyhedrons are constructed and accompanying ideas on how to teach these figures to elementary students. Also this article covered all of the various figures from cylinders to prisms giving a thorough background on how they are constructed and where they can be seen in real life. The usefulness of this article is quite understandable for this paper stresses the understanding of geometric figures and the need for the ability to think abstractly about geometric figures. This article also gives many encouraging thoughts on how to teach geometry to elementary students and the need to teach the correct application of geometry.

Pickreign and Capps (2000) presented very interesting explanations on how elementary school text books do not always line up with the National Council of Teachers of Mathematics' (NCTM) standards and principles for what elementary students should be learning. This article thoroughly stressed how children learn geometry and the need for the abilities to comprehend geometric concepts. This article also discussed the importance of geometry and how text books emphasize too much of one area while not enough of another. Also there is discussion on how geometry is presented more towards the end of most text books. This presents a problem for not all elementary classes are able to move through an entire text book in a single school year. From all of this though comes the idea of how geometry is taught and the need for certain aspects of

geometry to be included in every classroom. These aspects of geometry are ones the NCTM says are in the teaching of geometry but are not found in the text book. One of those aspects is the idea of spatial geometry and how it is not discussed enough in elementary text books. There is also discussion on methods of how to teach and reinforce these ideas that may not be presented in every text book as well. This article serves as great reinforcement for the idea of teaching spatial geometry. The many concepts and ideas behind teaching geometry worked excellently for the need of projects similar to the one's discussed in this paper. Teacher's must remember not all that is needed to be taught is found in a text book and it is important to stay knowledgeable on what the NCTM has established as important educational values.

Being able to identify various shapes is one of the most primary goals a teacher hopes their students will attain while studying geometry. Swarthout's (2003) article deals with the recognition of shapes in a picture, geoboard, on dot paper, etc. The project discussed in this article deals with having the students in a class create shapes using dot paper, geoboards, etc. to create the shapes they see. Then the students are to dissect their shapes and see what their shapes are made up of. It also encourages students to use different colors to identify the shapes they find within the shape they have created. This allows clear distinction between the new shapes making up the original figure, while adding an artistic element to the project encouraging the students even more to fully participate. While the students are working on identifying shapes they are also applying the knowledge they have previously learned making this project very useful in the full understanding of the makeup of various shapes. It is very important for students to understand and recognize what makes up a hexagon or a trapezoid. This article illustrates some very key elements to geometry which tie in very well with this paper. The most prevalent of those key elements to geometry is the ability to identify and understand the make up of geometric figures. This concept served as a great emphasis on the thorough education of geometry.

Kaufmann et al. (2000) stress ideas mainly concerning spatial abilities and their importance. First this article discusses the value of understanding the five components relating to the term spatial abilities. Those are: spatial perception, spatial visualization, mental rotations, spatial relations and spatial orientation. These five components to the term spatial abilities are very important in the education of geometry and the development of spatial skills. Though this article focuses much of its attention on 3D technology and virtual reality the overall concern for the understanding of spatial figures is always present. This article gives a great deal of detailed information on the use of technology in geometry. Programs such as Geometers' Sketchpad are a perfect example of the use of technology while studying geometry. Though Geometers' Sketchpad works with two-dimensional shapes only there are programs dealing with spatial shapes as well. The integration of technology in education is a great tool for students are always willing to "play" on the computer. Plus these computer programs allow a great deal of imagination when reshaping the figures and allow a student to see the various kinds of pyramids there are and their possibilities.

Overall these articles serve as a great background for the information discussed in this paper. The articles concern all the major areas this paper contains, such as learning techniques, learning geometric definitions, and understanding two-dimensional and three-dimensional geometry.

The following are three lesson plans that can be used in any classroom to help students understand polygons and polyhedrons.

Lessons that balance research with standards

Lesson 1

1. Assessing knowledge of plane figures.

A. Define plane figures.

1. What is a quadrilateral? Ask students to describe and draw a four sided shape. Many students will draw rectangles. Ask them to differentiate between a square and a rectangle. Also ask students to define a rhombus and a trapezoid. Make sure to show the differences and similarities between a square, a rectangle, a rhombus, and a trapezoid.

2. Define triangles. What is the basic component of a triangle? Name triangles by their sides and by their angles. Reinforce the idea that an equilateral triangle has three equal sides and all acute angles. This is review information as fifth grade students should have developed this information in fourth grade. (3.1.1: Students will identify, compare, define and classify polygons and circles.)

3. Review and reinforce the definition of additional plane figures.

- a. pentagon
- b. octagon
- c. hexagon

4. Stress and review the difference between regular and irregular polygons.

a. Remember that all regular polygons are equilateral and equiangular.

B. Have students carefully measure, cut out, and label 8 shapes using colored tag board.

1. These shapes should be as large or larger than a three inch by three inch square.

Lesson 2

A. Review and introduce polyhedrons by their sides and bases.

1. Polyhedrons discussed should be: Triangular prisms, triangular pyramids, cubes, and rectangular prisms.

2. Review and recognize the differences between prisms and pyramids.

Using models of both a prism and a pyramid have each student be able to differentiate between the two.

Review and discuss naming polyhedrons as described in the fifth grade text.

3. Have each student be able to recognize the differences between polyhedrons. i.e. how a cube differs from a triangular pyramid.

Begin cutting flexagon sides.

Form groups with the students and pass out multi-colored tag/poster board.

Pass out flexagon patterns to each group. Have them trace the patterns on to their tag/poster board and have them cut them out.

Stress the accuracy in cutting out the flexagons and how the corner wedges must be cut out correctly.

Assign cutting of simple shapes as homework.

- a. Those shapes should consist of triangles and squares.
- b. Each student should have a decent amount of both square and triangular flexagons.

Lesson 3

A. Review polyhedron shapes.

1. Have the students be able to clearly point out the various polyhedron shapes discussed in previous lessons.

B. Model flexagon construction.

1. Demonstrate to the class the basic flexagon constructions. i.e. cubes, rectangular prism, triangular prism, and a triangular pyramid.

- a. Show correct method of constructing a flexagon.

C. Have the students create and label their constructions.

1. Each student should be able to define their construction and explain its key features.

D. Creating a mobile with our polygons and polyhedrons.

1. Using a whole punch each student will need to make a whole in each of their polygons and polyhedrons.

2. Then each student should cut a piece of string for each of their polygons and polyhedrons, having each piece of string vary in size by a little.

3. Then take the coat hanger and tie the other end of the string to it.

4. Now each student should have a completed mobile and with the polygons and polyhedrons that they have created.

- a. Display the mobiles by hanging them over the desk of the student to whom they belong (see Figure 1)



Figure 1. Example of a Mobile with student created Flexagons

The lessons in this project thoroughly address the Montana state and national math standards. The standards from both sources state students in grades three through five need to know how to identify, compare, and analyze attributes of two- and three-dimensional shapes and develop a vocabulary to describe the attributes of these figures. Students also need to be able to classify these assorted shapes in accordance to their properties and build a distinction between the various classes of shapes. Also students need to understand the vast attributes of two- and three-dimensional shapes and be also to create them. Students should be able to create these shapes by drawing them as well as building them. Also students should understand three-dimensional objects are constructed from two-dimensional shapes.

The project displayed in this paper illustrates an excellent way to educate elementary students in geometry. This project also has the ability to speak to those students who are considered the “talented” or most exceptional in a classroom. The flexagons can range from an object as simple as a cube to a hexagonal prism. Also the arrangement of triangles and squares can produce very interesting shapes as well. Since there will always be students in a class who excel, it is important for a teacher to understand how to expand any project they work with. This project is a perfect example to work with because it can be expanded to incorporate all levels of intelligence. If a student believes or it seems what is being presented is too simple for him or her challenge them with a shape to build. Having the ability to increase the difficulty of a project is a great tool because the possibility to challenge any student in a classroom keeps them motivated.

I believe this paper demonstrates a logical and creative way of learning polygons and polyhedrons. Not only does this paper teach the students about polygons and polyhedrons but it makes them become involved. The project discussed in this article serves as a visual, verbal, and a kinesthetic form of learning while including and allowing multiple intelligence learning styles. For those students who learn best when presented with a problem on a page this problem fulfills those needs while also fulfilling the needs of those students who learn best with hands on projects. Not only can this project apply to students of various learning styles it also allows the students to be creative and use their imaginations making math more appealing and fun.

My hope with the project discussed in this article is to have students understand and learn how polygons and polyhedrons are constructed. This paper applies to strongly to teachers as well, and I hope teachers will benefit from the new knowledge of being able to present their students with a project that can include each student and allow them to explore geometry. I also hope students will see objects like polyhedrons are not limited to cubes and square pyramids, opening the student’s eyes to the vast and beautiful world of geometry.

References

Battista, M. (2002). Learning Geometry in a Dynamic Computer Environment. Reston, VA: The National Council of Teachers of Mathematics, 333-339.

Burke, K. and Dunn, R. (2002). Teaching math effectively to elementary students. *Academic Exchange Quarterly*. vol6, no.1, p91.

Kaufmann, H., Schmalstieg, D., and Wagner, M. Construct 3D: A Virtual Reality for Mathematics and Geometry Education (2000). *Education and Information Technologies*, p.263-267.

Koester, B. (2003). Prisms and Pyramids: Constructing Three-Dimensional Models to Build Understanding. *Teaching Children Mathematics*, pp. 436-442.

Oberdorf, C. and Taylor-Cox, J. (1999). Shape Up! *Teaching Children Mathematics* p340-346..

Pickreign, J. and Capps, L. (2000). Alignment of Elementary Geometry Curriculum with Current Standards. *School Science and Mathematics*, vol. 100, no. 5, 243.

Swarthout, M. (2003). Shape Search. *Teaching Children Mathematics*. vol.9, no.8, p455- 457.