ATTITUDE TOWARD MATHEMATICS: OVERCOMING THE POSITIVE/NEGATIVE DICHOTOMY

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1. Introduction

This contribution deals with 'attitude toward mathematics', a construct which plays an important role in mathematics education. Our interest in research on attitude dates back to several years ago.
In previous work (Di Martino & Zan, 2001, 2002, 2003; Zan & Di Martino, 2003) we underlined the lack of theoretical clarity that characterizes research on attitude and the inadequacy of most measurement instruments; we analyzed the definitions of attitude that either explicitly or implicitly researchers most frequently refer to; we discussed the 'negative' / 'positive' dichotomy; we found a relationship between the risk of circularity in research on attitude and the researcher's implicit beliefs.
In the following we will briefly summarize some results of these studies, in order to introduce recent findings from an Italian Project about attitude, aimed at investigating the phenomenon of 'negative attitude towards mathematics'.

1.1 The lack of theoretical clarity in research on attitude

Research on attitude has a long history in mathematics education. The construct finds its origin in the field of social psychology (Allport, 1935), in connection with the problem of foreseeing individuals' choices in contexts like voting, buying goods, etc. Research develops more toward the formulation of measuring instruments than toward the theoretical definition of the construct, producing instruments that have given theoretical and methodological contributions of great importance (such as those by Thurstone and Likert).

In the field of mathematics education, research on attitude has been motivated by the belief that *'something called "attitude" plays a crucial role in learning mathematics'* (Neale, 1969), but the goal of highlighting a *connection* between a 'positive' attitude and achievement has not been reached. Ma & Kishor (1997), after analyzing the correlation of attitude / achievement in 113 classical studies, underline that this correlation is not statistically significant: they explain this to be caused by the inappropriateness of the observing instruments that were used (in our opinion not only related to attitude, but also to achievement).
The attitude construct gains renewed popularity with the re-evaluation of affect in the learning of mathematics: in the classification of Mc Leod (1992) it is considered together with *beliefs* and *emotion* one of the constructs that constitute the affective domain (De Bellis & Goldin, 1999, propose *values* as a fourth construct).

Even if the meaning of the various terms is not always agreed upon, or even made explicit (Hart, 1989; Pajares, 1992) there is consensus on the fact that emotions and beliefs deeply interact: as regards attitude, an emotional component is generally explicitly recognised in the construct, often together with a cognitive component, mainly identified with beliefs.

Most researchers have underlined the need of some theory for research on affect, in order to better clarify connections among the various components, and their interaction with cognitive factors in mathematics education (Mc Leod, 1992). Recently the need for a theory for affect has been given several kinds of

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answers, differing for both the particular construct explicitly or implicitly chosen as 'starting' point (for example emotions or beliefs), and the different focus.[2]
Research on attitude has been judged to be particularly contradictory and confusing, due to the fact that it has given more emphasis to creating measurement instruments rather than elaborating on a theoretical framework (Kulm, 1980; McLeod, 1992; Ruffell, Mason & Allen, 1998): in actual fact, the work providing the most consistent information focused on the description of the *differences* between groups of people, usually males and females (Fennema, 1989).

1.2 Defining attitude toward mathematics
The lack of theoretical framework that characterizes research on attitude toward mathematics is partially shown by the fact that a large portion of studies about attitude do not provide a clear definition of the construct itself: attitude tends rather to be defined implicitly and a posteriori through the instruments used to measure it (Leder, 1985; Daskalogianni & Simpson, 2000).

When a definition is explicitly given, or can be inferred, it mainly refers to one of the three following types:

1. A 'simple' definition of attitude, that describes it as the positive or negative degree of affect associated with a certain subject. According to this point of view the attitude toward mathematics is just a positive or negative emotional disposition toward mathematics (McLeod, 1992; Haladyna, Shaughnessy J. & Shaughnessy M., 1983).
2. A multidimensional definition, which recognizes three components in the attitude: emotional response, beliefs regarding the subject, behaviour related to the subject. From this point of view, an individual's attitude toward mathematics is defined in a more complex way by the emotions that he/she associates with mathematics (which, however, have a positive or negative value), by the individual's beliefs towards mathematics, and by how he/she behaves (Hart, 1989).
3. A bi-dimensional definition, in which behaviours do not appear explicitly (Daskalogianni & Simpson, 2000): attitude toward mathematics is therefore seen as the pattern of beliefs and emotions associated with mathematics.

Kulm (1980) suggests that '*It is probably not possible to offer a definition of attitude toward mathematics that would be suitable for all situations, and even if one were agreed on, it would probably be too general to be useful*' (p. 358). In this way, the definition of attitude assumes the role of a '*working definition*' (Daskalogianni & Simpson, 2000). This position views the attitude construct as functional to the researcher's self-posed problems: in these terms we consider it to be useful in the context of mathematics education, as long as it is not simply borrowed from the context in which it appears, i.e. social psychology, but is rather outlined as an instrument capable of taking into account peculiar problems in mathematics education. This is in line with the position of Ruffell, Mason and Allen (1998), who see attitude as an observer's construct.

1.3 What does 'positive' or 'negative' attitude really mean?
In actual fact the term attitude is used in both practice and research together with the adjectives "positive" / "negative".
This dichotomy between positive/ negative attitude pervades mathematics education research, both implicitly and explicitly. For example, classic studies regarding the relationship between attitude and achievement in fact investigate the correlation between *positive* attitude and success. In the same way

[2] See for example the Research Forum about 'Affect in mathematics education: exploring theoretical frameworks' at PME 2004 (Hannula, Evans, Philippou & Zan, 2004), and the PME Special Issue with the same title (Evans, Hannula, Zan & Brown Eds., 2006). As regard beliefs, a considerable work of re-organization and systematisation has been done in the book 'Beliefs: A Hidden Variable in Mathematics Education?' (Leder, Pehkonen & Törner Eds., 2002).

studies aiming to change attitude actually end up in setting the objective of transforming a 'negative' attitude into a 'positive' one.

The definition of 'positive' or 'negative' attitude toward mathematics clearly depends on the definition of attitude itself.
According to the 'simple' definition, it is clear what a 'positive' or a 'negative' attitude is: a 'positive' attitude is a positive emotional disposition toward the subject; a 'negative' attitude is a negative emotional disposition toward the subject.
If we choose the bidimensional (or multidimensional) definition, it is not clear what a 'positive' attitude should mean, but referring only to the emotional dimension is reductive, since we have to take into account the two (three) dimensions, i.e. emotions, beliefs, (behaviours) and their interaction.
What actually happens is that in most studies the choice of a definition for attitude, and consequently a characterization of 'positive'/'negative' attitude, not only is not explicitly made: often it is not made at all, and the assessment / measurement instruments used by the researcher implicitly end up by continuously wavering between various definitions within a single study.
The characterization of an individual's attitude as positive / negative is in most cases simply the result of a process of measurement, performed through instruments such as the Thurstone or Likert attitude-scales or the semantic differential technique[3]. This process ends up in a score - attached to an individual's attitude - obtained by summing points relating to the single items. The choice of scores to be assigned to the items naturally leads to a positive/negative evaluation of each one.

Since in most questionnaires used to assess attitude the items range from those related to emotions ("I like mathematics") to those related to beliefs ("Mathematics is useful"), to those related to behaviour ("I always do my homework in maths"), an answer can be characterised as 'positive' by referring to different meanings of the word 'positive' itself. More precisely, this meaning varies depending on whether 'positive' refers to emotions, beliefs, or behaviour:

1) When it refers to an emotion, 'positive' normally means 'perceived as pleasurable'. So, anxiety when confronting a problem is seen as 'negative', while pleasure in doing mathematics is evaluated as 'positive'.

2) When it refers to beliefs, 'positive' is generally used with the meaning 'shared by the experts'.

3) When it refers to behaviour, 'positive' generally means 'successful'. In the school context, a successful behaviour is generally identified with high achievement: this naturally poses the problem of how to assess achievement (Middleton & Spanias, 1999).

In actual fact the three meanings overlap. For example, in the case of beliefs, sometimes 'positive' means that it is supposed to elicit a 'positive' emotion. A typical case is represented by the belief 'Mathematics is useful', which is also used in questionnaires aimed at measuring just the emotional dimension of attitude (i.e. the 'simple' definition of attitude: see Haladyna, Shaughnessy & Shaughnessy, 1983). But often 'positive' referred to a belief means that it is supposed to be related to a 'positive' behaviour, i.e. to a successful behaviour. Sometimes the latter meaning is also used for emotions, implicitly admitting that a 'positive' emotion toward mathematics, being pleasurable, is necessarily associated with a 'positive' behaviour in mathematics. On the contrary several studies (Evans, 2000) suggest the possibility that for certain subjects an optimal level of anxiety exists: above this level, but also below it, performance is reduced. The problem is that, generally differences between the various meanings are rarely made explicit: in this way, an a priori assumption is often made as to what should in effect be the result of an investigation, for example, that a belief which is 'positive' because it is shared by experts, is associated

[3] This characterisation is not so frequent in qualitative studies, and when it is used, it is generally accompanied by a description of the factors (behavior, beliefs, emotions) from which it is obtained. In any case, the evaluation of a positive attitude brings us back to a positive evaluation of at least one of the components: emotions, beliefs, behaviour.

with a 'positive' behaviour in that it is successful. This continuous sliding between the researcher's assumptions and the desired result of the investigation enhances the risk of *circularity* in research on attitude, a risk that Lester (2002) more generally pointed out for research on affect.
Depending on the criteria used to evaluate an attitude, different results may be obtained: for example, an attitude can be evaluated 'positive' as regards the emotional dimension, but 'negative' regarding the cognitive dimension, or vice versa[4]. The problem is only apparently overcome when the algebraic sum of the two components results in a single evaluation. Furthermore, as we have observed, beliefs are often used to assess the significance of the emotional dimension, or evaluated according to their 'behavioural' consequences, and this increases ambiguity.

The differences in the use of the adjective 'positive' not only imply different choices of assessment /measurement instruments: it also triggers a different formulation of the research problem to be dealt with. For example, the problem of identifying how to push a 'positive attitude', typically encountered in this field of research, requires a completely different approach depending on whether the positive attitude refers only to the emotional component or it refers to a particular pattern of beliefs and emotions, to be assumed as a model.

2. An Italian Project about attitude

The points made above about *the need for a theoretical framework for affect*, together with the importance of *linking theory and practice*, have been fundamental issues of an Italian Project about attitude. Several Italian researchers participated in the Project, named 'Negative attitude towards mathematics: analysis of an alarming phenomenon for culture in the new millennium[5]. The project's main objective was to investigate the phenomenon of 'negative attitude towards mathematics', viewed as 'something' that is connected to the learning of the discipline, but that also affects various aspects of the social context: the refusal of many students to enrol in scientific undergraduate courses due to the presence of exams in mathematics, a worrying mathematical illiteracy, an explicit and generalized refusal to apply rationality characterising scientific thinking, or, vice versa, a tendency to uncritically accept models that are only apparently rational.
One of the project's sub-goals was to provide an operative definition of 'positive' or 'negative' attitude toward mathematics, capable of giving teachers and researchers theoretical tools to observe and interpret some difficulties students meet in mathematics and possibly suggest ways to overcome these difficulties.
To reach this goal a theoretical reflection was needed, continuously and deeply linked to investigations about both origins of a negative emotional disposition towards mathematics and factors that influence its development. More precisely:
- a longitudinal investigation on a sample of students, covering the three-year duration of the project;
- collateral investigations performed on other subjects and involving teachers, students' family members, adults in general, and professional mathematicians.
The methodology entailed an integrated method approach:
- the use of questionnaires, diaries and interviews for the observation of teachers and adults;
- class observation, questionnaires, structured and semi-structured interviews, conversations, essays, etc. for monitoring students over the three years.
In the following, we will briefly present some results of this Project, that in our opinion can give a contribution to the theoretical issues discussed earlier.

3. Investigation on teachers' use of the 'negative attitude towards mathematics' construct

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[4] This is what Hannula (2002) observes, describing the evolution of the attitude of Rita, a lower secondary school student: he underlines that, using the term 'attitude' in a traditional manner, *'in the beginning Rita had an 'attitude' that was negative and positive at the same time'* (p.42).

[5] Besides the authors, the researchers participating in the Project are Pier Luigi Ferrari, Fulvia Furinghetti, Donatella Iannece, Paolo Lorenzi, Nicolina Malara, Maria Mellone, Francesca Morselli, Maria Polo, Roberto Tortora.

Within the Project, one of the activities conceived to favour a link between theory and practice has been an investigation carried out with teachers, aimed to reach two objectives:
1) to see whether in their practice teachers use the construct of negative attitude when they diagnose difficulty;
2) if this is the case, to see *how* they use it, investigating:
- what type of definition they make reference to (in particular, whether they use the 'simple' definition which sees attitude simply as an emotional disposition towards mathematics);
- if and how the diagnosis of negative attitude constitutes an instrument for intervening in a more targeted way on recognised difficulties.
These aspects were investigated through a questionnaire, administered to 146 teachers from various school levels: 29 from primary school, 50 from middle school, and 67 from high school (Polo & Zan, 2005). The questionnaire was specifically constructed to find out teachers' beliefs towards the negative attitude pupils can have towards mathematics, and contained 6 multiple choice questions and 6 open ended questions (see fig. 1). The multiple choice questions aimed at discovering whether and how frequently teachers use the 'attitude' construct in the diagnosis of difficulty, and if they consider changing a negative attitude at the end of high school a possible thing. The open ended questions intended to investigate what idea teachers have of 'negative attitude', and what indicators they use as reference.

Fig. 1: The questionnaire

School: _____
M ☐ F ☐ Age: _____ Date: _____

1. Do you ever find yourself attributing a pupil's difficulties with mathematics to his/her *attitude* towards the subject?
☐Yes ☐No
2. If yes, is this a frequent diagnosis or have you only seen it a few times?
☐practically never ☐sometimes ☐ rarely ☐often ☐nearly always
3. What do you mean by *negative attitude* towards mathematics?
4. What demonstrates to you that a student has a negative attitude towards
5. Do you think it is possible to modify the attitude of a pupil at the end of high school?
☐yes ☐ only to a certain extent ☐maybe ☐no ☐ don't know
6. If yes, *how*? If no, *why*?
7. Have you ever set yourself the specific objective of changing the attitude of one of your pupils?
☐Yes ☐No
8. If yes, how did you attempt to achieve this? What were the results?
9. Up to now we have only referred to a single student. Have you ever seen a negative attitude towards mathematics in a whole class?
☐Yes ☐No
10 How did you recognise this negative attitude?
11. If you answered yes to question 9, in this case did you explicitly set yourself the objective of changing the attitude of the class?
☐Yes ☐No
12. If yes, how did you try to reach this objective? What was the result?

The answers to Question 1 highlight the wide use of the term 'attitude' in relation to the diagnosis of a pupil's difficulty in mathematics: 85,6% of the sample (125 out of 146) in fact gave a positive answer to the first question. A comparison of the answers given by teachers from the various school levels shows a peak in the positive answers at middle school level (47 teachers out of 50, equal to 94%), with respect to the homogenous answers provided by primary and high school teachers (24 out of 29, equal to 82,8%, and 54 out of 67, equal to 80,6% respectively). The answers to question 2 confirm the frequency in the

use of the construct: 70 teachers out of 125 (equal to 56%) answer that they attribute a pupil's difficulties with mathematics to his/her attitude toward the subject sometimes / often / nearly always. A comparison between the various school levels highlights that the number of responses of that kind (sometimes/ often/ nearly always) increases with schooling level: 8 questionnaires out of 21 (equal to 38,1%) at elementary school level, 28 out of 40 (70%) at middle school level, and 34 out of 45 at high school level (75,6%).
The answers to the open ended questions give additional significant information:

- most teachers seem to have, even implicitly, the multidimensional idea of attitude, rather than the 'simple' one: they refer to students' beliefs about mathematics (in particular the belief that being quick in mathematics is very important, that mathematics is made of mechanistic rules, that mathematics is useless, difficult, ...), and to self-efficacy beliefs (students' claims such as 'I am not inclined to mathematics', 'Since primary school I always failed in mathematics');
- the teacher's apparently natural use of the 'negative attitude' multidimensional construct goes together with a lack of a clear distinction between the definition of attitude and the identification of indicators, thus making the definition itself not operative;
- the causes of a negative attitude are generally ascribed to *students'* characteristics and behaviours, thus hiding the teacher's responsibility in building a view of mathematics that elicits refusal, in the lack of interest and effort by students, in the image of the self that students construct;
- but most of all the diagnosis of 'negative attitude', referred to a single student, seems to be the final result of the teacher's interpretative process of the student's failure, rather than the starting point of a remedial action.

These results suggest that the attitude construct – as used by teachers – although rich enough to deal with the complexity of learning mathematics, does not seem to have the characteristics of a theoretical instrument capable of directing their work (particularly in that of helping students recover from difficulties): teachers rather seem to recognise a situation that is difficult to manage and modify.
In the end, this study highlights the importance of producing a definition of attitude toward mathematics, capable of making this construct a theoretical tool to direct teachers' observation, interpretation, remedial actions.

4. Investigating attitude through essays

In our Project the aim of giving a 'suitable' definition of attitude toward mathematics has been attained by observing students and collecting information through questionnaires, structured and semi-structured interviews, conversations, essays, etc..
One of the instruments used has been an essay on mathematics: "Me and mathematics: my relationship with maths up to now".
With this instrument we meant to get over the normative approach that characterises most research on attitude, and that we consider one of the reasons underlining both the lack of theoretical clarity and the difficulties encountered in getting significant results. Thus we adopted an interpretative approach, investigating students' relationship with mathematics 'from the bottom' and trying to spot in their descriptions the dimensions involved[6]. In other words, we did not assume an *a priori* definition of attitude toward mathematics, but we rather referred to the more general 'relationship with mathematics' in posing questions that underpin studies on attitude in mathematics education:

- How does attitude toward mathematics evolve during a subject's school experience?
- How can we explain the 'spoiling' of attitude toward mathematics from elementary school to high school?

[6] In the field of affect the need for social and anthropological approaches, i.e. for studying affect in its natural contexts, is particularly stressed, motivating the use of non-traditional methods, such as narratives (see for example Ruffell, Mason & Allen, 1998; Hannula, 2003).

- What are the variables that influence attitude toward mathematics? Which of them are controllable?
- Is it possible to modify a subject's attitude toward mathematics? How?

We ask the subjects to tell their own story with mathematics through an autobiographical essay: in doing this we are convinced that pupils will tend to explicitly evoke those events and remarks about their past that they deem important "here and now" and they will also tend to paste fragments, introducing some causal links, not in a logical perspective but rather in a social, ethical and psychological one (Bruner, 1990).
We assume that, in order to describe the kind of relationship an individual has with mathematics, this "pasting" process – typical of autobiographical narratives- is more important than an "objective" report of one's experience with the discipline at school. In other words, we agree with Bruner (1990)'s claim that it is not important whether the story told is actually «contradictory» or «likely»: we are rather interested in what the individual thinks he/she has done, the reasons underlying these actions, the type of situations he/she believed to be into and so on.

The essay was proposed at all school levels (for primary school the title was abbreviated in "*Me and mathematics*"): we explicitly asked that the essay be not proposed by the mathematics teacher. We collected a huge amount of materials, coming from different geographic areas and schools, but our sample cannot be viewed as a representative one, since it is based on both schools and teachers' spontaneous will to participate.
Overall we collected 1304 essays: 741 from primary school, 256 from middle school, and 306 from high school.

The analysis was carried out, according to an interpretative approach, trying to *understand* how students interpret their own experiences with mathematics, rather than to *explain* their mathematical path in terms of cause / effect. Final outcome of this analytical process is expected to be the construction of a set of categories, properties, relationships: a *grounded theory* (Glaser & Strauss, 1967), i.e. a theory based on collected data, the construction of which requires a continuous back and forth between the different research phases. In our case the essays were read in the light of both pre-existing categories (for instance liking and disliking mathematics) and in a free way, trying to identify meaningful categories *a posteriori*.

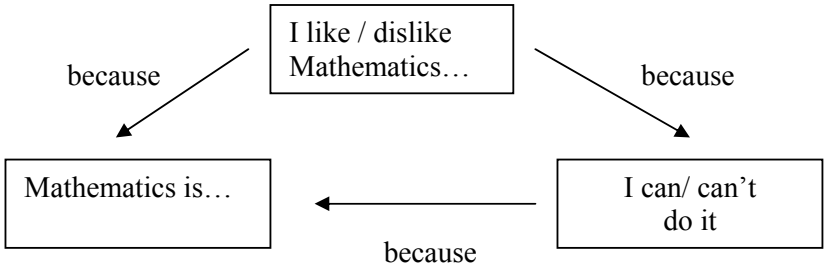
Although our type of sample requires a careful evaluation before any generalisation of data be made, significant points for analysis also come from quantitative-type reflections. Due to the huge amount of essays, we decided to use a specific piece of software for textual analysis, called T-Lab, to carry out this type of analysis.

5. Some results

As we mentioned earlier, we will only present here some results that can bring a significant contribution to the theoretical problems we dealt with so far.
Reading the essays, we identified three *core themes*, and precisely:
- the emotional disposition towards mathematics, concisely expressed with 'I like / dislike mathematics';
- the perception of being /not being able to succeed in mathematics, concisely expressed with 'I can do it / I can't do it';
- the vision of mathematics, concisely expressed with 'mathematics is…'.

The hypothesis suggested by a reading of the essays is confirmed by a quantitative analysis carried out through T-LAB: the most frequent expression in the about 1300 essays collected is 'I like' (in the different forms: I like / I don't like / I used to like …), followed by 'I can do it / I can't do it' and 'mathematics is …'.
Sometimes an essay develops around one of these three themes: more often, it makes reference to all the themes, although it is centred on one of them (which is therefore called the 'core theme' of the essay). It

is meaningful from both a theoretical and an educational perspective that these three themes are deeply connected: this clearly comes out from a reading of the essays and will be the basis for some remarks in the following.
The most frequent connection is associated with the word 'because': starting from the most recurrent theme 'I like / dislike', it is a motivation ('I like / dislike *because* …') that leads to one of the other two themes: the vision of mathematics or the perception of being / not being able to succeed. The vision of mathematics is also brought in when the core theme is the perception of being / not being able to succeed (I can / can't do it), once again through the underlying reasons: 'I can / can't do it *because* …'.



Let us examine these connections.

I like / dislike → vision of mathematics
This path highlights the fact that different emotional reactions (like / dislike) mainly link to different visions of mathematics
A widely spread distinction between these two visions of mathematics is that drawn by Skemp (1976) between one kind of mathematics characterised by 'rules without reasons' - named *instrumental* - juxtaposed to one vision of mathematics as *relational*, in which understanding means 'knowing both what to do and why'.
Often 'I like mathematics' is associated with a relational vision, and 'I dislike mathematics' to an instrumental one[7]:

I never liked to learn things by heart (except for some formulae) and this subject, together with Physics, gives me a chance to think and discuss. I like it, because it is a subject which needs reasoning. [3H.16]
I don't like it because there are many rules to make a tiny little operation: you must divide one number by the other one, take away the number you had before and so on. Moreover, if you forget a rule you run into troubles! [1M.16]

Sometimes the instrumental vision of mathematics is associated with a positive emotional reaction:

As I was growing up I got more and more fond of mathematics, because I starter to do expressions (the things I prefer in this subject); and so, after seeing that I was starting to get them right, then mathematics or, better say algebra, became my passion. [2H.74]

In the end, sometimes one same vision of mathematics can be associated with different emotional reactions. This is the case for the characterisation of mathematics as 'rational':

[7] In the following excerpts the first number refers to the class level, the letter refers to the school level (Primary / Middle / High), the last number indicates the progressive numbering of the essay within the category .

It is fascinating because it is not an opinion, it is a rational subject (like my own character), which needs no interpretation; … it is so. [5H.4]
(…) this does not mean that I like mathematics, actually I completely hate it, simply because it is a subject I feel really far from me. When you have to solve an equation you don't need to be creative, to interpret or say what you feel; mathematics is empty of feelings, just think of the well-known saying: "mathematics is not an opinion". [5H.1]

To conclude, these data stress the fact that the two dimensions - vision of mathematics and like/dislike - are mutually independent.
This independence is strongly expressed in characterising mathematics as useful/useless, and easy / difficult. These dichotomies are particularly interesting to our discussion, since the items "Mathematics is useful" (with a positive score to an affirmative answer) and "Mathematics is difficult" (with a negative score to an affirmative answer) are frequently used in questionnaires about attitude.
Reading the essays we have found all four combinations of like/dislike and easy/difficult:
I like mathematics because...
… it is easy: *Since primary, I always found mathematics rather simple and easy to be understood. And maybe this is the reason why it has always been one of my favourite subjects.* [2H.73]
… it is difficult:
The story of mathematics in my life started off with logic: sets of fruits, difference between "and" and "or", opposites (like red and not red) […]. As time was going by, as fruits disappeared and notions and difficulties were increasing, mathematics became more and more interesting and involving and my judgement changed completely. [5H.16]
I don't like mathematics because...
… it is easy: *In the beginning mathematics was nasty because it was too easy: 3+1, 5+5,...* [4P.115]
… it is difficult: *I didn't like mathematics much because I saw it was difficult and I did not manage I gave up quickly.* [1H.36]

As regards the useful/useless dichotomy, essays very often show that even when mathematics is perceived as useful, this perception is not necessarily associated with a positive emotional disposition:
Although it is a useful subject, I don't like it. [1H.3]

I like / dislike → I can / can't do it
The connection beween the theme 'I like' and the theme linked to self-efficacy, expressed through 'I can / can't do it' comes out so strong from the essays that sometimes the expressions "I like" ("I dislike") and "I can do it" ("I can't do it") are used as synonyms:
Since primary school, I remember when the teacher asked us to number by 2, 3, 6, 9 up to 800, 900 … I used to hate it. Then I changed school and I started to hate it even more because of the expressions. Let's not talk about middle school I changed 4 teachers in the 3 school years and therefore if I didn't understand anything before, now I really understand zero. [1H.3]

In the greatest part of the essays in which the dimensions 'I like / dislike' and 'I can / I can't do it' are connected, this connection is *"I like it because I can do it"* and *"I dislike it because I can't do it"*. In some rare cases, we found the combinations *"I like it although I can't do it"* and *"I can do it but I dislike it"* .
From both an educational and a theoretical perspective, the most interesting outcome of the reading of the essays is that 'success' in mathematics has many deeply different meanings. In some essays 'succeeding' is identified with school success, i.e. with getting good marks, and thus it is up to the teacher to acknowledge one's success. In some other cases, 'succeeding' is identified with 'understanding'. In the latter case things are come complicated: sometimes 'understanding' is used with an *instrumental* meaning, and it is identified with knowing the rules and being able to apply them correctly; in other cases a

relational-type 'understanding' appears, referring to one's awareness of why the rules work and how they are linked to one another.

I can/ can't do it → vision of mathematics

When perception of being / not being able to succeed ('I can / can't do it') is the core theme of the essay often the pupil talks about the reasons underlying his/her success or failure explicitly: these are the so called *causal attributions* for success and failure (Weiner, 1974). These attributions often allow us to recognise not only the pupil's beliefs about him/herself but also his/her vision of mathematics, sometimes through the so called 'theories of success', i.e. the theories a pupil may have about success in mathematics. Once again a wide range of theories emerges, corresponding to a wide range of ways of viewing mathematics: for instance, theories of success or attributions that centre on the important role played by memory, suggest an instrumental vision of mathematics, whereas theories of success and attributions focusing on the need to understand what one is doing, suggest a relational vision of mathematics:

Up to middle school I have always succeeded in mathematics, because I always understood the reasoning behind it. [1H.15]

It is not that I don't understand maths, it's that I make a mess, because it's full of rules and theorems and it's almost impossible to remember each of them, moreover when I'm finally at ease with a topic, it seems they make it on purpose: we carry on with the planned contents and I am cheated. [2H.20]

6. Conclusions

The results presented here confirm some issues we discussed in the beginning.

The choice of using agreement on certain beliefs to deduce a positive emotional reaction is questioned by the fact that different emotional reactions (like/dislike) can be associated with one single vision of mathematics. This is particularly true for two widely used items: "Mathematics is useful", generally viewed as 'positive', and "Mathematics is difficult", generally viewed as 'negative'.

Analysing the collected essays we observed that the majority of them develop around two recurrent and generally interconnected core themes: *I like*/*dislike* and *I can/can't do it*. These themes are intertwined in the author's *vision of mathematics*: in particular, one single emotional reaction can be associated with conflicting visions of mathematics, and possibly conflicting emotional reactions can be associated with different visions of mathematics.

The essays describe pupils' relationship with mathematics as referred to three dimensions: like/dislike, perception of being / not being able to succeed, vision of mathematics. In the sample we examined these three dimensions are combined in many different ways.

These results suggest that for a description of a pupil's attitude towards mathematics it is not enough to highlight his/her (positive / negative) emotional disposition towards the discipline: it is necessary to point out what vision of mathematics and what self-efficacy beliefs this emotional disposition is associated with.

If we start from this more complex definition for attitude, a problem arises about what should be meant by 'positive' or 'negative' attitude. Referring to the only emotional component seems to have limitations from both a theoretical and an educational perspective: this would lead us to define as 'positive' the attitude of a pupil who views mathematics as a discipline made of rules to memorise and apply rigidly, only because he likes the subject itself.

It is thus possible to introduce different types of 'negative' attitudes, different 'profiles of attitude', depending on the component the adjective 'negative' refers to. It might be referred to a distorted and epistemologically wrong vision of mathematics, and as such 'negative', as in the example discussed above. Rather, it might be referred to the 'beliefs about the self' component, outlining as 'negative' those beliefs about the self which are characterised by a scarce sense of self-efficacy ("I'm not able", "I can't make it", "I'm hopeless").

Some recurrent patterns can be traced in the diverse combinations emerging from the essays. For instance, in essays telling a story of difficulties or unease, an instrumental vision of mathematics often emerges,

together with negative emotions, identification of success and achievement, theories of success that stress the role of memory, scarce sense of self-efficacy. The recurrence of these combinations make them significant from an educational point of view: the teacher is thus provided with hints on possible routes to be followed to enable pupils to *narrate a different story*.

After all, this way of seeing attitude towards mathematics – constructed as a grounded theory – may become a useful instrument for both teachers and researchers. The diagnosis of a 'negative' attitude becomes a starting point for the teacher to design an intervention aimed at modifying the component(s) identified as 'negative' for that pupil. As for researchers, this definition, as it has been constructed, provides a strong link with practice, prevents from falling into circularity and, in the end, allows them to overcome some of the critical points of research on attitude.

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