# MODELLING IN SCHOOL- CHANCES AND OBSTACLES

**Hans-Wolfgang Henn**[1]
**University of Dortmund**

**Dedicated to Günter Törner's 60th birthday**

**Abstract**: *One goal of mathematics education is to educate pupils to become responsible citizens. Important for this is the achievement of modelling competence of students at school. I want to identify chances and obstacles which can promote the development of these competences, but can also obstruct them drastically. This will be illustrated by concrete examples.*

## The Relevance-Paradox

The American fantasy writer Terry Pratchett is very popular among young people in English speaking countries as well as in Germany. One of his scenarios is a "discworld". This reminds at once of the novel Flatland by Edwin Abbott which was published in 1884 and of the many possible mathematical aspects in such a scenario. This might lead one to believe that Pratchett was also interested in mathematics. In the preface of the German edition of one of his books (Pratchett, 2005) he writes, however "I want to emphasize that this book is not in the least insane. Such a description is only apt for sappy mathematicians, who mix up geometry with the joy of living." (my translation).

Pratchett is not alone with this attitude. Mathematics, especially in German society, is seen as something which is no fun and has nothing to do with real life. My friend Mogens Niss (1994) relates this to the *Relevance Paradox*. This means that mathematics is entering deeper and deeper into more and more parts of life, but people do realize this less and less.

From my point of view, mathematics has two important sides. On one side mathematics is a special science with a special culture of thinking. Mathematics has its own aesthetics and beauty, which, however, is not accessible to everyone, same as with literature, fine arts, or music. One the other side, mathematics possesses an extraordinary functionality that allows us to bring order and understanding to all parts of our life, but also bears the danger of misuse. The aim of mathematics education needs to be to help students acquire a coherent view of mathematics. Teaching needs to be oriented towards *both* aspects and should be able to help students experience *both* of them.

Apart from elementary calculation skills the central aim of mathematics education is to acquire abilities to select appropriate methods for the application of mathematics in different areas of life. Related to this is an understanding of the phenomenon *mathematics* as cultural achievement and product of human thinking. This means that special mathematical content and details, which might be useful for students in their lives after school are of less importance compared to methods and possibilities of mathematics, because actually more than 99% of our students will not work as mathematicians in their further life.

[1] Prof. Dr. Hans-Wolfgang Henn
Institut für Entwicklung und Erforschung des Mathematikunterrichts
Fachbereich Mathematik, Universität Dortmund
Vogelpothsweg 87, 44227 Dortmund
Tel: +49 231 755-2939 (Sekr. -3745, FAX -2948) E-Mail:
wolfgang.henn@uni-dortmund.de
URL: http://www.wolfgang-henn.de

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**Mathematics and the Rest of the world**

The brilliant methaphor of "Mathematics and the Rest of the World", that can be presented by the well-known picture of the modelling circuit (Fig. 1) originates from Henry Pollak (1979).

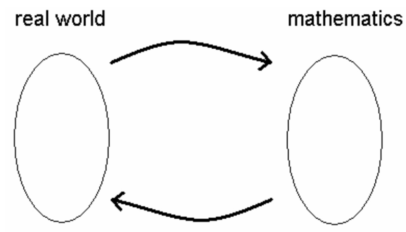



Fig. 1: The Model Circuit

An application of mathematics requires a relation to the world beyond mathematics. One needs to understand something better, explain or predict certain phenomena, solve problems, provide foundations for decisions and so on. These can be a questions from science, a technical applications or a problem with a private, social or political background.

Relations between reality and mathematics are especially relevant within the PISA framework (compare Neubrand, 2003). There, the construct of „mathematical literacy" was defined as

> "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage in mathematics, in ways that meet the needs of that individual's life as a constructive, concerned, and reflective citizen."

This concept aims directly at the future life of young people. In PISA the use of the mathematical knowledge of students is analysed within various situations and contexts of reality.

The importance of applications of mathematics and mathematical modelling is shown by the study initiated by the ICMI, the *International Commission on Mathematical Instruction*, on this topic. Recently, the Study Volume belonging to this study, *Modelling and Applications in Mathematics Education* (Blum et al., 2007), was published.

**What are models?**

Models are simplifying presentations, which consider only certain, somehow objectifiable parts of reality. A simple example is a map. Models are mappings from reality into mathematics. The purpose for a model is to draw conclusions for reality. Of course, for a model to be good, you must show it leads somewhere. This crucial point that models need to be helpful for something, that conclusions need to be drawn for reality and that these conclusions need to be tested in an *experimentum crucis* is often forgotten in so-called application problems in school.

Especially important are the following two aspects of models: On the one side, we have *normative models*. Examples are the law for the income tax, the methods for elections, the rules for the soccer world championship, and so on. On the other side, we have *descriptive models* which we differentiate between *models which predict* (for example the weather forecast), *models, which explain* (for example why do we see a rainbow), and *models, which describe* (for example the development of HIV).

The HIV example demonstrates how difficult it is to extrapolate from given data. Fig. 2 shows data for HIV new infections in Germany from 1979 to 1983. This picture suggests a dramatic increase in numbers of HIV infections. Instead, as Fig. 3 shows, the number of new infection decreased visibly after 1983, owing to improved precaution.

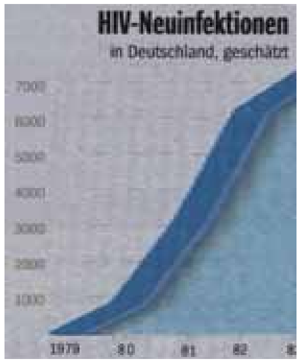



Fig. 2

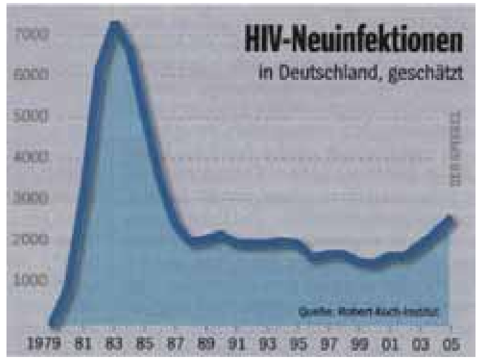



Fig. 3

Models for a real problem can be more or less suitable. One should never talk about “right” or “wrong”. For example, it does not make sense to call Newton’s model of physics “incorrect” and Einstein’s model “correct”. Both models provide a reasonable description of nature under the given circumstances. There are “hard” models such as the quantitative models of physics which use model assumptions as the law of gravity. These models allow excellent predictions. Rather “soft” models such as models in economics or ecology, on the contrary, are often overvalued. In each case, models will always be of subjective character, owing to the normatively chosen assumptions. This also includes the danger of misuse and misinterpretation. It is an important task for school to impart knowledge about this fact.

**Basic experiences in Maths Education**

Following Heinrich Winter (1995/2004), the well-known German mathematics educator, three “basic experiences” are necessary in order that mathematics lessons will convey general education principles on every school level. Those basic experiences are

(BE 1) to realise in a specific way, and to understand phenomena in the world around us, which we are and should be concerned with,

(BE 2) to learn about and to understand mathematical issues represented in language, symbols, pictures, and formulas as intellectual creations, as a deductively ordered world of its own kind, and

(BE 3) to acquire problem-solving (heuristic) skills by analysis of tasks which go beyond mathematics.

The first basic experience points out the fundamental contributions of mathematics towards acquiring important knowledge about our world. The second one aims at the “inner world of mathematics”, mathematics shows that a rigorous science is possible. And finally the third one describes mathematics as a school of thought. For example, “problem solving” (Henry Pollak: “Here is the problem, solve it”) is inherent to the last basic experience.

Mathematics proves to be an inexhaustible pool of mathematical models, which allow us to understand better the world around us. However, for concrete examples, both of the two other basic experiences play central roles, too, especially when the important demand for interconnectedness according to the spiral principle is taken seriously. It is clear that our theme, modelling, relates to BE 1. In any case, applications and modelling are an indispensable part of the basic experiences of Winter and thus contribute to conveying a balanced image of mathematics in school at every level.

**Modelling: Chances and Obstacles**

Unfortunately, as a rule, reality-oriented teaching on applications outside mathematics is covered only to a limited extent in every day teaching although there is a long-standing agreement on the importance of creating relations between realistic situations and mathematics teaching.

I will especially point out three important factors which can promote the development of modelling competence of pupils, but can also obstruct them drastically. These are

- the problem field „central exams“,
- the use of computers, and
- the professional development and the motivation of the teachers.

***The problem field "central exams"***

Central examinations have a crucial influence on the content of teaching. Therefore, central examinations can also influence positively and foster the important relations between mathematics and "the rest of the world".

But often the so called application problems in central exams are only mathematical problems 'in disguise' and not genuine real life problems. For the students 'uncovering' these problems 'in disguise' is reduced to finding out the algorithms that have been hidden by the teacher, and immediately 'real' mathematics takes over. Those exams problems are more or less so called 'age-of-the-captain' problems. This kind of problems was first discussed from Stella Baruk (1989), such as the following:

There are 26 sheep and 10 goats on a ship. How old is the captain?

Joe, a primary school pupil, picks both numbers and argues: $26 - 10 = 16$, that's the age of my brother, but he is too young to be a captain. $26 \cdot 10 = 260$, my grandpa is only 85, so that cannot be, too. $26 : 10$ is not possible to calculate, so there is only one possible correct answer for the wanted age: $26 + 10 = 36$.

We laugh, but central exams often have the same impact: centralized assessment for final examinations or even tests at the end of each school year often reduce teaching to mindless drill and practice of procedures and calculation techniques. The following example is taken from the central final examination (Year 13) of the German state of Baden-Wuerttemberg, posed in 1998 in the topic Analytic Geometry: The problem is set in the context of a playground with a wooden pyramid that stands perpendicularly on a square base and is accessible inside.

The next text shows part c of the problem (my translation):

> Inside the pyramid a board is fixed parallel to the floor with a circular opening with a diameter of $d = 2.4$ in its middle. For tidying up, a big foam ball with radius $r = 1.5$ needs to be pushed through the opening towards the upper part of the pyramid. At which height is the board fixed if it is supposed to be as high up as possible with the ball lying loosely in the opening?

The missing measurement units show immediately that the problem poser does not take reality too seriously. Nowadays, it seems perhaps necessary to point out the importance of tidying up in school. Indeed, one puts more and more educational goals on the shoulders of the teachers... Back to the Abitur task: We assume that the measures are given in meter and make a drawing of the situation (Fig. 4).

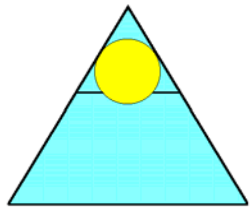


Fig. 4

The board is fixed at a height of 5.6 meters, and the ball possesses a volume of 9.4 cubic meters. The internet states the specific weight of foam: the ball weighs approximately 380 kg! How should this ball ever be pushed upwards? How should it ever be pulled out again? Maybe the problem poser was thinking about a giant screw pull like in Fig. 5, to add the corresponding geometric helix curve to the problem?

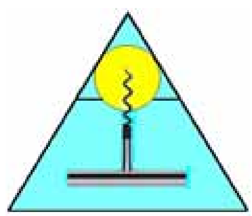


Fig. 5

Anyway, the problem is a typical 'age-of-the-captain' problem and such problems, given in central exams, influence teachers in their belief that modelling and applications are meaningless for mathematics teaching.

### *The use of computers*

Today's available computer technology can contribute in a special way to aid in the learning process, and is equally helpful and important for all three basic experiences (see Henn, 1998). Firstly, the computer is a powerful tool to aid in modelling and simulation. Secondly, the computer can positively influence the generation of adequate basic concepts ("Grundvorstellungen") of mathematical ideas – especially through dynamical visualisations. Lastly, the computer furthers heuristic-experimental work in problem solving.

But the computer does what you want – sensible or foolish. The following problem, in my opinion, belongs to the last category. I found it in an American journal for didactics of mathematics (Martinez-Cruz & Ratcliff, 1998). The authors investigate the men's world record times in 100m freestyle swimming from 1912 to 1994. Without any mathematics, just using common sense, one would expect qualitatively something like the curve in Fig. 6.

This qualitative curve has nothing to do with the modelling assumptions of logistic growth. For the intermediate time there are no reasonable model assumptions pointing at a special curve.

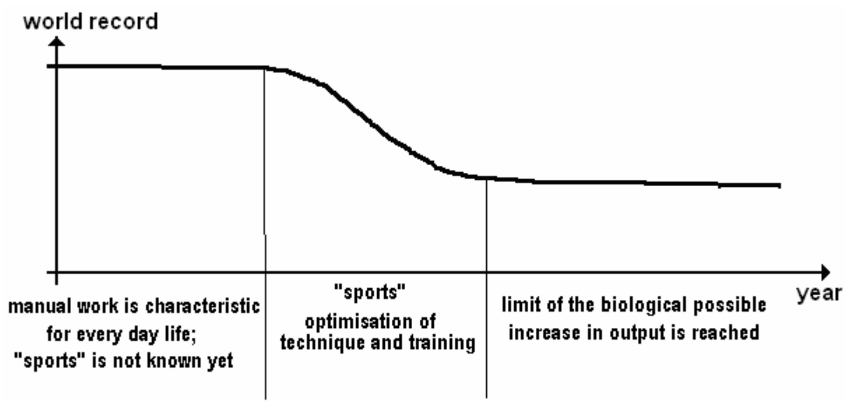



Fig. 6

The authors use the following table (Table 1) of the world record times and fit various curves through the given record data by applying the regression commands available on their calculator.

| *Year* | *Time (sec)* |
|---|---|
| 1912 | 61.6 |
| 1924 | 57.4 |
| 1957 | 54.6 |
| 1968 | 52.2 |
| 1972 | 51.22 |
| 1976 | 49.99 |
| 1988 | 48.42 |
| 1994 | 48.21 |

Table 1

In detail, they fit a linear function y = a·x+b, an exponential function $y = a\cdot b^x$, a power function $y = a\cdot x^b$, and a logistic function $y = \frac{c}{1-e^{b+ax}}$. The Fig. 7 shows that the choice of the curve is irrelevant for the interval in question. However, extrapolation on both sides in Fig. 8 shows that all models do not represent the real situation.

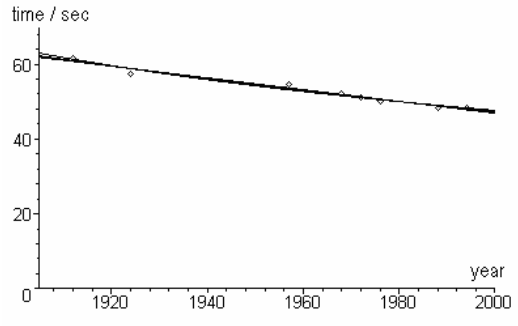



Fig. 7

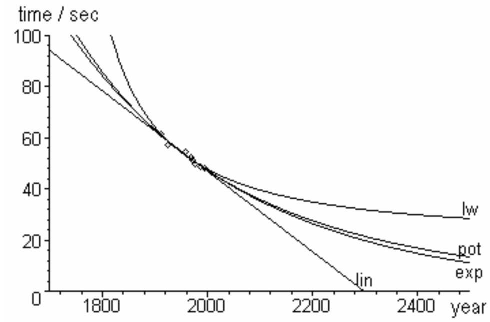



Fig. 8

The authors favour the logistic model, because its predictions are less meaningless for the future compared to the other predictions! Naturally, the logistic curve – as often in occurrences from nature – is similar to the qualitative curve form one would expect, but it provides neither any explanation nor any meaningful prediction outside the measured data. No one of the four models provides deeper insight. The correlation coefficients of all four models do not differ significantly and are all nearly 1!

By the way, the authors do not consider one of the most interesting points of the table: it is the increase in measurement accuracy from 1968 to 1972. In 1972, the Olympic Games took place in Munich. At first, times were taken with three digits after the decimal point, an accuracy of 1/1000 second. This can be reconstructed from the results of the 400 m medley swimming contest (Table 2):

| | | |
|---|---|---|
| **400 m medley swimming** | 1. Gunnar Larsson (Sveden) | 4:31.98 OR |
| (30.8.1972) | 2. Tim McKee (USA) | 4:31.98 |
| | 3. Andras Hargitay (Hungary) | 4:32.70 |

Table 2

At first, times were recorded as 4:31.981 for Larsson and 4:31.983for McKee and therefore, Larsson was awarded the gold medal. Then, obviously, somebody started thinking: It takes about 50 seconds to swim 100 m, that means a distance of about 2 mm in $^{1}/_{1000}$ second. Nobody would believe that a 50 m long swimming pool could be constructed so accurately that each swimming lane had an accuracy of less than 2 mm. A little bit more mortar already leads to a larger difference. Therefore, the measurement accuracy was reduced to two digits after the decimal point. But, incomprehensibly no two gold medals were awarded!

### *The professional development and motivation of teachers*

It is an important task to educate teachers to include applications and modelling in their teaching practice. This implies to „see the world with mathematical eyes“ and to find occasions, again and again, to introduce some situations from reality in the mathematics classroom. A simple way to do this is to use newspaper clippings. However, caution must be applied not to overshoot the mark.

The following two newspaper clippings from different newspapers (from Herget & Scholz, 1998, my translation into English) denounce one of the often meaningless regulations which are issued from the German Federal Government.

Clipping 1:

**Perfect official language**

> The perfection of German makers of regulations was documented by Secretary of the Interior Georg Tandler in the parliament in Munich by reading out the draft for an ordinance on hold-

> ing calves in Germany. It is written there, however that might mean in plaintext: "If calves are held in herds each calf has depending on its withers height in centimetres to have a freely usable space in square meters according to the following formula: Minimal space (square cm) equals 0.4 times to the power 2 plus 70 times plus 2,720."

Clipping 2:

**Authority Mathematics**

> "If calves are held in herds each calf has depending on its withers height in centimetres to have a freely usable space in square meters according to the following formula: (mathematical exponential way of writing) minimal space cm (to the power of) 2 equals 0.40 x (to the power of) 2 plus 70 x plus 2720." (Taken from the new draft of the German federal states for an ordinance on holding calves.) The CDU-member of the parliament of the German state of Hessen Dieter Weirich (Hanau) suggested to translate this draft to farmers and to give official help for computation. One needs to ask, commented Weirich, if farmers facing such "droppings from public offices" will ever get around to muck out their stables.

The two texts have mainly the same content, but there is one mathematically interesting difference in the way how each of the two authors describes the functional term:

1^th^ text: "… following formula: Minimal space (square cm) equals 0.4 times to the power 2 plus 70 times plus 2,720."

2^nd^ text: "… following formula: (mathematical exponential way of writing) minimal space cm (to ´the power of) 2 equals 0.40 x (to the power of ) 2 plus 70 x plus 2,720."

Only the second journalist understood the mathematical symbol x correctly as the variable for the height of the calf, the first one reads it as the symbol for multiplication and gives a totally meaningless text. But even from the second text it is not easy to develop the correct formula. I gave the task to grade 9 students who developed about 6 or 7 formulas and wrote the down on the blackboard. Finally, we agreed on the following one.

$f(x) = 0.40 \cdot x^2 + 70 \cdot x + 2{,}720.$

After the graph has been drawn (Fig. 9) one can reflect on sense and nonsense of this regulation. One of my students immediately argued: It is much simpler to use a straight line!

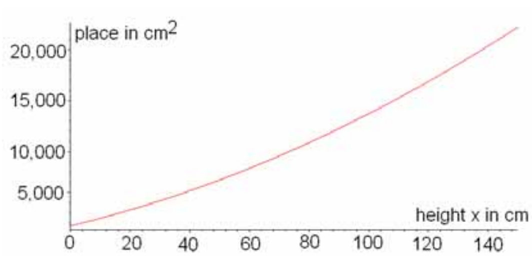



Fig. 9

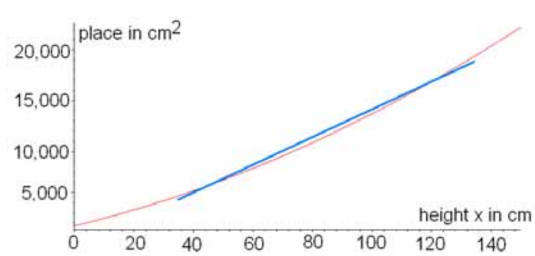
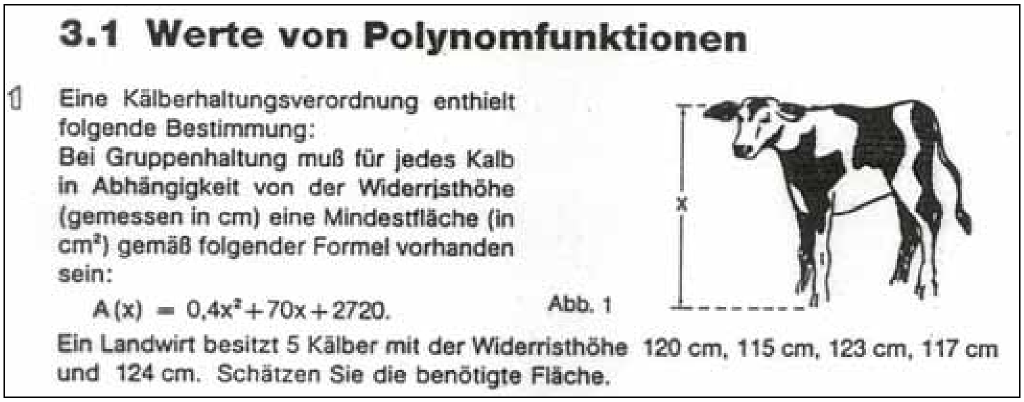



Fig. 10

The bold line in Fig. 10 would give the same result but would lead to a simpler regulation.

So far, so good! But, even this nice problem can be put into bad teaching practice out of sheer enthusiasm about applications and modelling. This is illustrated by the following two examples:

The first example is this schoolbook problem on the calf regulation in Fig. 11 (from Sigma, 1984).



**3.1 Werte von Polynomfunktionen**

1 Eine Kälberhaltungsverordnung enthielt folgende Bestimmung:
Bei Gruppenhaltung muß für jedes Kalb in Abhängigkeit von der Widerristhöhe (gemessen in cm) eine Mindestfläche (in $cm^2$) gemäß folgender Formel vorhanden sein:

$A(x) = 0{,}4x^2 + 70x + 2720.$

Abb. 1

Ein Landwirt besitzt 5 Kälber mit der Widerristhöhe 120 cm, 115 cm, 123 cm, 117 cm und 124 cm. Schätzen Sie die benötigte Fläche.



Fig. 11

The topic is “Werte von Polynomfunktionen” (values of polynomial functions). At first, I will translate the task 1 into English:

> An ordinance on holding calves contained the following provision: “If calves are held in herds each calf has depending on its withers height (measured in centimetres) to have a freely usable space (measured in square meters) according to the following formula:
>
> $A(x) = 0.4x^2 + 70x + 2720.$
>
> A farmer has 5 calves with withers heights 120 cm, 115 cm, 123 cm, 117 cm and 124 cm. Estimate the necessary area.

Without reflection, the regulation is cited, the term “Widerristhöhe”, that means the withers height of the calves, is explained using a drawing with the variable x, instead having children search for explanations for themselves. Then, without any comment, the formula A(x) = … is given. The task is now to substitute five values in the formula and to add. This is not the way to develop a reality-oriented problem, but a typical age-of-the-captain problem.

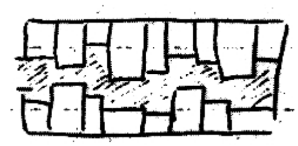
The second example is the following part of the mathematical diary of a girl (my translation into English). A young teacher, more correct, a teacher trainee, had covered the calves problem adequately in the classroom and now the girl reports on this:

I owe the second page to Mrs. Koch, a teacher trainee.

The problem was set in the context of calculating the necessary space in a stable for a calf of size x. Maybe this was meant to broaden the pupils' horizon for the unlimited possibilities to use functions. In this case the function increased exponentially, which would mean that the farmer needed to apply a straightedge regularly to find out about the growth of each of the calves and then to assign them a new, bigger place in the stable. I would argue in favour of a minimal value that would make any calculation superfluous.

However, according to Mrs. Koch, a linear function would turn out to be an indispensable help for the farmer, because he could read off the necessary space comfortably from a graph.

I do not agree to this. How would his life be made easier, if his stable needed to look like this?



Of course, the graph does not "increase exponentially" as the girl mentioned, but that is not the point. The point is that the teacher, out of pure enthusiasm, gave the impression that a regulation with a linear formula would be the only reasonable solution. The girl proved to have more common sense than the teacher.

## Modelling in school

Mathematical modelling is the mutual fertilization of mathematics and the rest of the world (Pollak, 1979). Through modelling, students are enabled to build a bridge between mathematics as a tool to understand better the world around them, and mathematics as abstract structure. For this, suitable teaching situations are indispensable. Lyn English (2003) demands "rich learning experiences", i.e. authentic situations, chances for own exploration, multiple possibilities for interpretations, and social competence to take up the responsibility for one's own model up to communicating it to other students. Teachers are often reluctant to include mathematical modelling in their teaching. Katja Maaß (2006) points out that the complete modelling process is time-consuming and difficult. She also shows, however, that modelling activities can be started successfully in a normal teaching situation. Students should not be spared the difficulties and effort related to applications and modelling. I will demonstrate this with some examples.

### *Who needs numbers, has the choice: Ideal, real and computer numbers*

From my point of view, the number line is the most important model in school. Meaningful use of numbers is an indispensable skill for educated citizens. In life, in and after school, numbers happen to occur in three forms: There are the ‚ideal' numbers of mathematics, for which obviously

$2 = 2.0 = 2.00$

applies, and the real numbers from daily life, which often turn out to be intervals, for which 2 will not be the same as 2.0. Mostly, for example when measuring, intervals, not exact numbers are an adequate model for the situation. Intervals, however, will lead to error propagation in further calculations. If this is forgotten, results can become arbitrary very quickly. On top, nowadays we also have the computer numbers, which lead a life of themselves. While the performance of processors has increased rapidly, the error analysis of the implemented floating point arithmetic has been neglected immensely.

A quick estimation illustrates the rounding problems of numerical calculations: The mathematical model of the reflection law states for each angle of incidence $\alpha$ always the exact value $\beta = \alpha$ of the angle of reflection (Fig. 12).

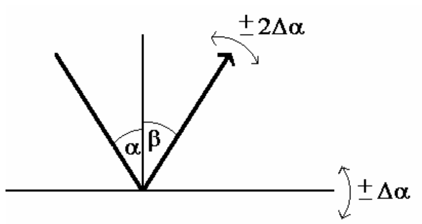



Fig. 12

But, if the position of the reflector is only known with a deviation of $\pm\Delta\alpha$, then the angle of reflection $\beta$ is given by the interval $[\alpha-2\Delta\alpha; \alpha+2\Delta\alpha]$, the error has doubled. After n reflections, the error has increased to $\pm 2n\cdot\Delta\alpha$ . Even with a very small initial error of $^{1}/_{1000}$ degree the error has increased to a value of more than 360° after 18 reflections – no predictions can be made any more.

### *Examples from practice for practice*

The following few examples show that “seeing the world with mathematical eyes” will provide many occasions to motivate modelling activities in the classroom.

Maintenance house

Fig. 13 shows a maintanence house along a railway line. It is not clear, how the distance indication can be given exactly in meters. The picture provides a good starting point to discuss the use of numbers which should be suitable to describe a real situation.


Fig. 13

USA Eagle Stamp

In a publication for the European market the US postal administration listed the size of the Eagle Stamp (Fig. 14) released on April, 29th, 1985, to be 48.768 x 43.434 millimeter.


Fig. 14

One little fault is that the measurement unit should be square millimeters. Again meaningless is the exactness of the measures. Already a change of humidity will change the size of the stamp more! What is the reason? In the United States the size of the stamp was listed to be 1.92 x 1.71 square inches. The person responsible for converting the size to metric measurements for the European market simply multiplied with the conversion factor of 2.54 and forgot to round meaningfully. A further example for lacking numeracy!

Mount Everest and other mountains

While touring Britain we travelled near Ben Nevis, the highest mountain in Scotland. The guide said she had no clue how one could determine the height of this mountain. Again and again it is amazing that even educated persons have no idea how to do this, even if intercept theorems and trigonometry are taught in school. Memorized formulas and the application of mathematical theorems to concrete application problems are two different things. Activities doing concrete measurements outside with self-built measuring instruments should be an integral part of teaching in the middle grades.

There is another mathematically interesting story about the measurement of the height of the highest mountain, the Mont Everest (Fig. 15).

Fig. 15

Around the middle of the 19$^{th}$ century British surveyors took measurements on the height of the mountain from different places (Poindexter, 1999). Taking the arithmetic mean from the different measures in 1852, by chance they got a result of exactly 29,000 British ft. They did not want to publish this result as they worried this would be taken as an imprecise measurement with an accuracy of only ±1,000 ft. Therefore, they published 29,002 ft as the height of the Mount Everest.

Small sugar bags

In restaurants one gets sugar in various packages (Fig. 16). Of special interest are the bags provided by Burger King (Fig. 17), which have an imprint stating that “45 % less paper than usual packing” is used.

Fig. 16

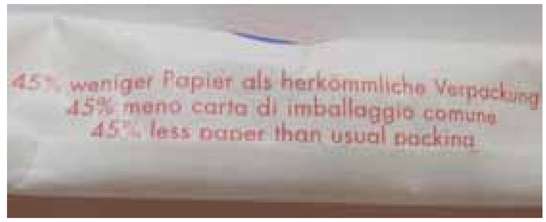



Fig. 17

Secondary school students can investigate and judge on this statement using various methods.

**Conclusion**

Teaching affects the image that students will take with them into their future life as responsible citizens and future decision makers. This image should contain both the beauty *and* the functionality of mathematics. But applications of mathematics in other fields should not be studied for its own purpose alone. Reflecting on what relates mathematics with the rest of the world is indispensable, ethical issues of mathematical actions have to be highlighted, and students have to be sensitised for it.

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