

MATHEMATIZING DEFINITIONS OF BELIEFS

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A Review of Törner, G. (2002). Mathematical beliefs—a search for a common ground: Some theoretical considerations on structuring beliefs, some research questions, and some phenomenological observations. In G. C. Leder, E. Pehkonen, & G. Törner, (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 73-94). Dordrecht, The Netherlands: Kluwer Academic Publishers.

With the incisive logic of the mathematician he is, Günter Törner examined definitions of *belief* in the current literature on the role of beliefs in the teaching and learning of mathematics, noted the lack of consistency in usage and focus, and set out to structure and systematize key aspects of these definitions. In doing so, he accomplished more than a systematization of the literature in this significant area of influence in mathematics education. Characteristically, he introduced his own mathematical symbolism in this analysis, and in this way attempted to mathematize a phenomenon of human experience—that of *believing*—that some may consider to be a far cry from the certainty and predictability of the logic of mathematical principles. What did he accomplish in this mathematization? How may the results of his work be used to enhance understanding of the role of beliefs of various kinds in mathematics education? Does his model provide a theoretical framework for further research on this significant topic? These and related questions are addressed in the following review of Günter Törner’s accomplishments in his chapter in the book on *Beliefs: A hidden variable in mathematics education?* that he edited with Gilah Leder and Erkki Pehkonin.

Introducing mathematical terminology and symbolism in an attempt to systematize definitions of beliefs

The style of writing in this chapter is succinct, yet lucid and rational. Working out the core elements of a definition of beliefs is a major goal, as summarized in Törner’s own words as follows.

Specifically, a four-component definition of beliefs is presented. The model focuses on belief object, range and content of mental associations, activation level or strength of each association, and some associated evaluation maps. This framework is not empirically derived but is based on common characteristics of the literature on didactics, particularly mathematics didactics. This effort towards achieving a precise definition can provide new understandings of fundamental issues in research on mathematical beliefs and give rise to new research questions. In particular, it allows description of the term “belief systems” allowing clustering of individual

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beliefs into a system across each of the four components. Furthermore, it makes sense to distinguish between global beliefs, domain-specific beliefs and subject-matter beliefs. (p. 73)

In each of the four components of the definition, mathematical symbolism is introduced. This symbolism is surely a means of keeping track of the complexity of the human phenomenon of *believing*, but it is also a possible means of deepening the issues that are the foci of attention in research in this field. Before introducing each component, Törner builds a rationale for his decisions and stance, and in the process demonstrates an impressive familiarity with relevant literature. For instance, he notes that previous work has not adequately distinguished between knowledge and beliefs, but he does not pursue this avenue because “beliefs can be viewed as at the periphery of knowledge (Ryan, 1984)”, as he later elaborates in his model.

Quoting Fischbein, Törner notes that “Beliefs are not mere residuals of more primitive forms of reasoning. They are genuinely productive, active ingredients of every type of reasoning” (p. 75). Further, beliefs filter the processes used in establishing the “virtual realities” in our mental processing of sense data. The profound conclusion of this observation is that in the subjective nature of this processing we do not all see the same phenomena, even when observing the same things: *We see what we believe!* The implications of this insight for mathematics teaching and learning are tremendous. However, Törner does not pursue this aspect in the chapter.

The four components of a belief, **B**, are as follows.

1. There is a belief object, **O**, because it is always a belief *about something* (p. 78). Further, there is a breadth dimension, a **range of associations**, which makes up the content set, C_O , related to the object O. Some of the elements of the set C_O may be conflicting (p. 79).
2. Because C_O is a *fuzzy set*, some elements of the set are central and others more peripheral. Thus it is necessary to specify the *range and content of mental associations* by means of a **membership degree function**, μ_i , whose values vary from 0 to 1 (p. 79), i.e., $0 \leq \mu_i \leq 1$. One advantage of this notation is that it incorporates the distinction of beliefs from knowledge. Knowledge would consist in a set C_O in which the μ_i functions are equal to 1 for all values of i .
3. The **activation level or strength of each association** may also be invoked by special membership degree functions, μ_i , with values again varying from 0 to 1. These functions may measure
 - a. levels of certitude;
 - b. levels of consciousness; and
 - c. levels of activation (pp. 80-81).
4. Affective evaluations of beliefs take place (as implied in many definitions). Thus one or more **evaluation map(s)** ϵ_j are defined for the range of a belief C_O and with a linguistic value scale that may be bi-polar or continuous, e.g., degrees of liking or disliking (pp. 81-82).

It is also necessary to keep in mind that there is a **person P**, who has professed the belief or to whom the belief is attributed, and that this phenomenon took place at a certain **time t of constitution** (p. 82). For a person P at a time t, Törner summarizes the situation as follows.

In short, a belief constitutes itself by a quadruple $B = (O, C_O, \mu_i, \epsilon_j)$, where O is the debatable belief object, C_O is the content set of mental associations (what traditionally is called a belief), μ_i is the membership degree function(s) of the belief, and ϵ_j is the evaluation map(s). (p. 82)

Although the foregoing definitions may be precise in their mathematical terminology, they are not closed, because they take into account the complexity of human variations in experience, at various times, within certain contexts, and above all they allow for the variability of human thinking and reacting to experiences. The formulation also allows for the clustering of beliefs according to the four domains in the definition, resulting in *belief systems*, and the tripartite further distinction of beliefs that are global, domain-specific, or subject-matter specific (pp. 85-87). Various hierarchies are possible in this organization. Törner perceives that “in many contexts it is not sufficient to study beliefs; the analysis of belief systems must take priority” (p. 84).

Implications for research on beliefs in the teaching and learning of mathematics

The foregoing gives some indication of what Törner accomplished in his mathematization of a definition of beliefs. The possibilities are now open for the results of his work to be used to enhance the teaching and learning of mathematics. Such enhancement is likely to come through the avenue of research that uses his lucid categories as theoretical lenses with which to define research parameters and analyze research results, in the investigation of beliefs of various kinds in mathematics education.

Although the main thrust of this chapter is theoretical, in a final broad section before his conclusions, Törner offers “some qualitative observations” (p. 88) with regard to beliefs and their role in attitude changes. In this section he also gives readers an illustrative glimpse into his own research on mathematical beliefs. In particular, six preservice upper-secondary school teachers were asked to write freely about their experiences with calculus lessons in response to the prompts, “How I experienced Calculus at school and university”, “How I would have liked to have learned Calculus”, and “How I would like to teach Calculus” (p. 88). Analysis of the data yielded seven types of beliefs related to the teaching and learning of calculus. In this illustrative account of the study, two students’ beliefs regarding the role of logic in calculus are contrasted: for one it is a barrier to be overcome, for the other it is something far more foundational, involving elements of epistemology. One insight as a result of this investigation is that it is necessary to consider domain-specific beliefs (such as those expressed by these preservice teachers) in terms of global views of the nature of mathematics itself. In research on mathematical beliefs, ontology and epistemology are intertwined.

Törner does not indicate how the lenses of his mathematized definition of beliefs helped the analysis in the research reported. The use of his framework in research is left open. It may be that the purpose of the mathematization of the definitions was not to provide a re-usable theoretical model for research, but for the precise clarification of the components themselves. However, what he has accomplished may be gauged in comparison with some results in the history of mathematics that he introduced earlier in the chapter (p. 77), where he noted that the proliferation of terms used by various authors instead of *beliefs* contributed to the lack of consistency in the literature. The very openness of the field may not be an impediment in and of itself. He pointed out that a similar situation existed with regard to beliefs about the nature of number in the 19th century: however, work on number theory continued, and Dedekind (1985/1888—mistakenly given as 1988 in the text) was moved to write about “What are numbers and what should they be?” This analysis led the way to an axiomatic definition of numbers. In a comparable way, Törner’s careful and logical work to compose components of a definition of beliefs has the potential to further the field of research on the role of beliefs in mathematics education, whether or not mathematical symbolism is employed in using the components of his model. It is possible that his careful work may lead to a finer grain of understanding of some aspects of mathematics education, and in this way contribute to improving the teaching and learning of mathematics at various levels.

This is a very balanced chapter. The careful and at times profound analysis of components of beliefs and their systems was appreciated by this reviewer. Because of the rigor and clarity of his approach, Törner has provided a theoretical model that deepens our insights into the complexities entailed in processes of

human believing. In doing so, he opens new possibilities and starts a new conversation in this field of considerable significance for mathematics education.