THE ROLE OF BELIEFS ON FUTURE TEACHER'S PROFESSIONAL KNOWLEDGE

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Abstract: *In this paper qualitative results of a complimentary case study embedded in a large-scale study on future mathematics teachers' professional knowledge will be presented. Referring to the classification system discussed within the didactical scientific community we study the beliefs of future mathematics teachers about mathematics and the teaching and learning of mathematics as well as concerning future teachers' professional development. Based on interviews, the beliefs of two selected future teachers are contrasted. From this, the central role of education of pedagogical content knowledge on the development of beliefs during teacher education becomes evident.*

"Doppelte Diskontinuität" – Double Discontinuity

Der junge Student sieht sich am Beginn seines Studiums vor Probleme gestellt, die ihn in keinem Punkte mehr an die Dinge erinnern, mit denen er sich auf der Schule beschäftigt hat; natürlich vergißt er daher alle diese Sachen rasch und gründlich. Tritt er aber nach Absolvierung des Studiums ins Lehramt über, so soll er plötzlich eben diese herkömmliche Elementarmathematik schulmäßig unterrichten; da er diese Aufgabe kaum selbstständig mit seiner Hochschulmathematik in Zusammenhang bringen kann, so wird er in den meisten Fällen recht bald die althergebrachte Unterrichtstradition aufnehmen, und das Hochschulstudium bleibt ihm nur eine mehr oder minder angenehme Erinnerung, die auf seinen Unterricht keinen Einfluß hat- Felix Klein[2]

At the beginning of his teacher education, a young student faces problems which did not remind him at all on what he has worked on when he was at school; of course he forgets those things quickly and completely, but, if then, after he has passed his exams, he starts working as a teacher, he shall suddenly teach exactly this elementary mathematics in a school adequate manner. As he is nearly unable to connect this elementary mathematics to his university mathematics, in most cases, he will soon come back to the old-fashioned teaching traditions and his university education remains more or less a romantic memory which has no impact on his teaching.

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[2] From: Felix Klein (1933), Elementary mathematics from a higher perspective. Vol. 1 (*Elementarmathematik vom höheren Standpunkte aus. Erster Band*.) p. 1.

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Dedication: *This contribution is devoted to Günter Törner on the occasion of his 60th birthday. Günter Törner has demonstrated an impressive broadness in his research work, and, besides many others, one of the main emphases of his research he has been on beliefs in mathematics and mathematics learning for which he often refers to the "Double Discontinuity" described by Felix Klein already at the beginning of the 20th century, which prevents real changes in mathematics teaching. In this paper we refer to approaches that have been developed together with Günter Törner and we hope therefore to make them fruitfully applicable in teacher education.*

Introduction

Teacher education has already been criticised for a long time without its effectiveness ever being analysed empirically on a broader base. There are only a few empirically based results about the impact of the worldwide varying teacher education systems on future teachers' knowledge and their developing of competencies (for an overview about the actual situation of empirical research see Blömeke 2004). Therefore, the International Association for the Evaluation of Educational Achievement (IEA) started an international comparative study in 2006, taking the education of mathematics teachers of the primary and lower secondary level as an example (Teacher Education and Development Study: Learning to Teach Mathematics – TEDS-M). By this study, for the first time the effectiveness of different teacher education systems were analysed worldwide. Concerning the future teachers, the aim of TEDS-M was to record data about their knowledge in three areas, in mathematics, mathematics pedagogy, general pedagogy and psychology. Beyond that, the future teachers were asked about their beliefs and their personality traits. Furthermore, institutional and curricular analyses were conducted, for which the intended and implemented curriculum of the educational institutions were recorded. The results of this study, which are being prepared by 20 countries (including USA and Germany), are expected to be out by December 2009.

Due to the problems with the conceptualisation of the theoretical foundation, on behalf of IEA a preparatory study has been conducted in six countries, the "Preparatory Teacher Education and Development Study (P-TEDS)", also participated by USA and Germany. As part of this study, at the University of Hamburg several supplementary and more detailed case studies on professional knowledge of future teachers and the influence of beliefs have been carried out. First results will be presented and discussed in this article. This supplementary study concentrates exclusively on the micro level of P-TEDS, the level of individual competence acquirement and the influence of beliefs, limited to the first phase of teacher education within a consecutive structure of teacher education.

Theoretical framework of P-TEDS and TEDS-M and the objectives of the supplementary case study

The initial ideas of P-TEDS und TEDS-M are considerations about the central aspects of teachers' professional competencies as basically defined by Shulman (1986) and developed and differentiated further by Bromme (1994, 1995) and others (about discussions on theoretical professional foundation see Blömeke 2002). TEDS-M and P-TEDS are based on the conceptions of professional competencies of future mathematics teachers elaborated by Bromme and Weinert, and they are aimed at receiving data about the requirements for the professional tasks of future teachers, such as teaching and making diagnoses. Furthermore, by referring to Shulman (1986), the following three knowledge areas are distinguished:

(1) Mathematical content knowledge sub-divided into:

- The required cognitive activities of future teachers, based on fundamental ideas of mathematics such as algorithmising or modelling;
- Mathematical content areas such as algebra or statistics;
- Mathematical levels, i.e. school mathematics of lower secondary level or upper secondary level, school mathematics from a higher point of view and mathematics at university level;

(2) Pedagogical content knowledge in mathematics sub-divided into:

- Mathematical content areas as under point (1);
- Teaching-related tasks of mathematics teachers like elementarisation of mathematical concepts or the diagnosis of students errors;
- Stimulated cognitive activities of students, including amongst others problem solving or modelling in everyday life situations.

Referring to Bromme (1994) knowledge of mathematical pedagogical content knowledge (called by Bromme as subject-matter-specific pedagogical knowledge) is understood as the central field where mathematical content knowledge, general conceptions about mathematics, knowledge about curricular conceptions of mathematics teaching and aspects of teaching experiences as well as knowledge about the students perceptions interweave each other.

(3) General pedagogical knowledge focussing on teaching and diagnostic questions.

(4) Professional competencies comprise of affective and value-oriented aspects apart from cognitive-oriented dimensions of knowledge measured via belief components. These aspects will be differentiated according to the following belief categories:

- beliefs about mathematics as a scientific discipline;
- beliefs about teaching and learning mathematics;
- beliefs about teaching at school and learning in general;
- beliefs about teacher education and professional development.

Concerning mathematics-related *beliefs,* we refer in our complementary study to differentiations of Grigutsch, Raatz & Törner (1998) as it is done similarly by the P-TEDS-main study. They differentiate *beliefs* according to four categories that proved to be applicable for our complementary study. The above listed categories are characterised by statements about how mathematics is perceived. Basically there are:

- The formalism aspect characterises mathematics by a pure logic and formal proceeding and lingual precision. The deductive character of mathematics is put into the foreground,
- The schematic aspect which regards mathematics as the application of solving routines. Mathematics is understood as a conglomerate of special rules, formula and procedures;
- The application aspect regards mathematics as being useful in everyday life and emphasises the applicability and problem solving abilities;
- The process aspect which states among others that mathematics is felt as being intensively heuristic and as creative activity and that problem solving can be done in various individual ways by practicing mathematics.

The formalism and the schematic aspects are categorised as a static approach to mathematics and learning and teaching mathematics while the application and process aspects are categorised as dynamic approaches.

(5) Personality traits in a professional and non-professional context

As these aspects cannot be considered completely within the framework of the following case study, we will not discuss them further. Within the framework of this study we do not include personality traits as well as further kinds of belief. We are focussing more on the so-called epistemological *beliefs*, subjective thoughts about the nature of mathematics and the characteristics of teaching and learning mathematics (see Törner 2002). In order to focus on the role of beliefs in teacher education, we will not give detailed descriptions on the other components of teacher knowledge. Especially differences between P-TEDS and

TEDS-M are not considered, thus we refer to the framework of TEDS-M and publications in preparation on P-TEDS (see Tatto et al. 2007, Blömeke, Kaiser, Lehmann 2007). Similar conceptualisations of professional knowledge of mathematics teachers are used in other studies as well. We particularly refer to the COACTIV study, which basically deals with questions about the conceptualisation and the measurement of subject-based professional knowledge of mathematics teachers and possible correlations with students' development of achievement. (see among others Brunner et al. 2006a and the overview of Baumert, Kunter 2006). However, further conceptualisations of professional knowledge exist: Thus Ball, Hill, Bass (2005) distinguish among common knowledge of school mathematics and specialised knowledge of teaching mathematics and of mathematics acquired through professional training, but due to lack of space, we cannot go into detail here.

Within the scope of P-TEDS, we have conducted detailed in-depth case studies in order to analyse future teachers' professional knowledge and its relations to beliefs of which a few results will be described in the following. The case study is focussing on future teachers and their first phase of teacher education.

Methodical approach

Within the framework of a number of additional – qualitative-oriented – studies to PTEDS a questionnaire with open items has been developed that concentrates on the areas 'modelling and real world context' und 'argumentation and proof' and admits more qualitative analyses. This questioning has been conducted with 80 future mathematics teachers on a voluntary base within the scope of pro-seminars and advanced seminars for future teachers at the University of Hamburg. The questionnaire consists of 7 items that are domain-overlapping designed – as so-called 'Bridging Items'. Each of the items captures several areas of knowledge and related beliefs; 3 items deal with modelling and real world examples, 3 with argumentation and proof and one is about how to handle heterogeneity when teaching mathematics. Furthermore, demographic information like number of semesters, second subject and attended seminars and teaching experiences – including extra-university teaching experiences - are collected (for first results see the supplementary study by Kaiser, Schwarz 2007 and contributions in Blömeke, Kaiser, Lehmann 2007).

Based on this questioning, 20 future teachers – participating also on a voluntary base – were asked more details by means of problem-centred guided interviews (Witzel 1985) about the above named topic areas. The guideline for the interviews contains pre-structured and open questions (ask-back questions). The interviews lasted about 45 to 90 minutes, and, in order to avoid fear of failure and feelings of assessment, future teachers instead of professors or university lecturers conducted the interviews. For the case study presented below, two contrasting interviews were chosen which proved to yield interesting results. The audio taped interviews were transcribed verbatim and then evaluated according to qualitative content analysis methods (Mayring 1997), more exactly, we referred to methods of the structuring content analysis aiming at extracting a specific structure from the material by using criteria that have been defined before. This means that by referring to definitions, to typical passages of the text functioning as so-called anchor examples as well as to encoding rules, an encoding guideline is developed to analyse and to structure the material. Our main content categories were defined in a theory-guided way and oriented towards beliefs as described above. For the category system differentiations we referred to the classification system of beliefs developed by Grigutsch, Raatz, Törner (1998) as described above in chapter 1.

Then, by means of the developed category system, the interviews and the questionnaires were structured and encoded. A generalization of the results is neither intended nor possible due to the small sample size. However, the results permit exemplary insights into possible structures of professional knowledge of future mathematics teachers but limited to the area of modelling.

Case descriptions and first results of the study

In the following two future teachers (names are changed) are introduced shortly. They are described based on the results of the interviews and the questionnaires.

Anne is studying for becoming teacher at the primary and lower secondary level in the 3rd semester. Her second subject is history. Until the time of the interview she has attended the basic lectures in mathematics, one lecture in mathematics pedagogy and several seminars in general pedagogy. She is giving private lessons regularly.

Ben is close to the final examination and aims at becoming a teacher in vocational schools. He had already gained a diploma in economics before his teacher study. His second subject besides mathematics is economics. Until present, Ben has attended numerous lectures on mathematics and mathematics pedagogy and carried out two practical trainings in school.

(1) The case of Anne

We start by presenting Anne's beliefs about mathematics as school subject or scientific discipline:

With Anne static as well as dynamic based epistemological beliefs can be reconstructed while static oriented beliefs are dominant. She states:

> *„…also wenn ich an Mathe denke, dann ist das sehr vielfältig, halt mit Zahlen und Formeln umgehen, Gleichungen umstellen, diese ganzen Rechenarten…, knobeln, so ein bisschen was man auch in den Heften machen muss…"*
>
> *„… so, if I think on maths it is a many folded thing, handling numbers and formula, reorganise equations, all these operation methods, tossing, a bit of those things one had to do in the exercise book…"*

To her, mathematics means abstract systems of formulae, equations and calculation rules dealing with the inversion of equations, for which one must apply formulae and know arithmetic rules. However, the fact that she also notices tossing indicates a dynamic aspect that is inhered in mathematics.

However, her epistemological beliefs are also characterised by a formalism aspect which becomes obvious when dealing with the role of proofs in mathematics and school mathematics.

> *„… es gehört einfach elementar zur Mathematik dazu, so ist die gesamte Mathematik aufgebaut, dass nach und nach in den ganzen Jahren und Jahrhunderten irgend welche Sachen bewiesen wurden, so dass man sie verwenden durfte und das muss man zumindest mal gehört und gesehen haben, wie so was funktioniert. Das gehört einfach dazu."*
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> *„…it is an elementary element of mathematics, all of mathematics is structured like that, that over all the years and centuries things were proved so that one was allowed to use them and that one has at least once heard about it or seen how it functions. That's simply part of it."*

Anne is still at the beginning of her teacher education, her more traditional and static *beliefs* about mathematics are hardly surprising (see Blömeke, 2004). However, first signs of dynamic perceptions of mathematics could be observed with Anne, possibly due to her actual experiences during her study and her practical work. Thus, as objectives of mathematics teaching she states:

> *„Und auch so ein bisschen was neben Mathe, also dass Mathe eben auch für den Alltag gebraucht wird, nicht nur für den Matheunterricht."*
> *„And a bit besides maths, that maths is also used for everyday life, not only for math lessons."*

According to this statement, for Anna, mathematics does not only mean a formal handling of numbers and equations etc. but for her mathematics is also strongly connected with everyday life. Thus, mathematics is in a way also a tool for solving everyday life questions and problems.

To resume, the majority of her statements reflect that Anna has predominantly static understanding of mathematics. Obviously, she refers to many of her own experiences when she was at school, such as doing specific formalisms (e.g. "given", "search for", "solution"), gesucht, Lösung) which regards important or rigid schematic ways of thinking when doing proofs. All in all, in her opinion formalisms (formal proofs etc.) from university study in mathematics should be integrated more into mathematics lessons at school because she expresses that her school knowledge did not provide her the knowledge she needed for her university study. These more 'traditional' *beliefs* are partly contrasting with dynamic perceptions of mathematics which emphasise a stronger relation to reality and the real world.

From her *beliefs* about learning and teaching mathematics, both perceptions of mathematics can be recognised. On the one hand we find receptive thinking of teaching and learning as can be seen from the following statement:

> *„Ich mache als Zweites noch Geschichte und da ist es fast noch wichtiger als in Mathe, dass man da einen didaktischen Aufbau hat, weil man die Kinder ja nicht einfach rechnen lassen kann, ein paar Aufgaben, oder so, sondern man muss da durchgehend immer Methoden wechseln."*
>
> *„As second subject I am studying history and there it is nearly more important than in maths that one has a didactical structure because one cannot let the children simply doing calculations, just some tasks, or so, but you must constantly change the methods."*

This shows that to her didactic structures of mathematics lessons are less important, because mathematics lessons mainly engage students in solving arithmetic problems.
These beliefs of Anne about learning and teaching mathematics are strongly related to static and formalism views on mathematics as described above and which are also expressed by Anne's statements about the meaning of proofs:

> *„…ich finde es schon wichtig, dass sie von vornherein lernen, dass man Sachen beweisen kann, dass man es nicht nur zeigt, sondern dass da auch was Allgemeines hinter steckt."*
>
> *„… I think it's quite important that they [the students] learn from the very beginning that one can prove things, that one does not only show it but that there is something universal behind it."*

The statement about proofs shows that to her teaching means strongly pure knowledge transfer which tells students how things are. However, the teacher shall not leave it at that but show that mathematics is characterised by formal proofs.

> *„Für die Schule denke ich sollte man sich so langsam steigern. Am Anfang wird den Kinder ja eh nur gesagt, so ist das […] und dann langsam dieses anschauliche, präformale Beweisen und dann sollte man aber irgendwann schon auch zum formalen Beweisen kommen. Damit die Kinder das auch mal gesehen haben."*
>
> *"For school I think one should increase gradually. At the beginning children are just told that something is like that […] and then slowly these descriptive, pre-formal proofs and then, after a while, one should come to the formal proofs. So that children have also seen it."*

This strongly receptive conception of learning is in contrast to a constructivist statement where she emphasises the necessity of teamwork for modelling tasks:

„…also nach dem Beispiel hab ich schon gesehen, dass es eben Schüler gibt, die das völlig falsch angehen könnten. Und darauf hab ich gedacht, das ist immer ganz günstig, wenn sie einen Partner haben, mit dem sie sich absprechen können. […] Einfach als Hilfestellung, dass sich auch keiner allein gelassen fühlt, weil er gar nicht weiß wie er es machen soll.“

„…After looking at that example, I have seen that there are students who start with it in a totally wrong way. And after that I have thought that it is always profitable if they have a partner with whom they can discuss. […] Just as assistance so that nobody feels left alone because he does not know how to do it.”

It becomes clear that she prefers active learning and teamwork in groups to individual learning. In this context it is important to let all students participate in the learning process and to support the own solution attempts of the students.

„Und dass man die Kinder auch dahingehend unterstützt, dass man sagt, ja das ist ein toller Ansatz und mach das mal weiter. […] wenn man die Kinder vorher schon lobt, oder wenn sie auch wissen, dass sie was Richtiges machen, dann trauen sie sich auch.“

„ And that one helps children that way that one says ‚that's a good approach, just go on". […] if one compliments children for something or if they also know that they are doing something correctly, they are more encouraged.”

This ambivalence between the procedure of prescribing solutions and approaches and encouraging to do something in one's own way is a red thread running through the whole interview as the following statement shows exemplarily. There she describes how she would act, if learning difficulties occur:

„…oder dass ich einfach Mal etwas vorgebe, dass ich sage, rechnet doch das mal aus und dass sie dann verstehen, […]. Und dass sie dann vielleicht auf die Idee kommen das weiter zu führen.“

„… or that I just give something in advance, that I tell them to calculate it so that then they will understand […]. And that then they will get the idea to continue with it.”

Her beliefs about mathematics teacher education and professional education of future mathematics teacher strongly emphasise the necessity of a solid subject-related education as essential basis of teacher education:

„…das Fachwissen ist, denke ich, einfach unabdingbar. Es muss auf jeden Fall da sein, denn wenn ich das Fachwissen nicht hab, kann ich auch die Didaktik nicht lernen, das ist so ein bisschen von einander abhängig…“

„…the subject-related knowledge I think is indispensable. It must exist, because if I don't have that subject-related knowledge I also cannot learn the didactics, they depend a bit on each other.”

However, basically, she argues that at primary and lower secondary level the emphasis is put on the children while at the upper secondary level on science.

„Ich möchte halt wirklich mit Menschen arbeiten und ich hab mich nur schwer getan auf Oberstufe zu studieren, allein der Mathematik wegen und hab mich dann aber für die Kinder entschieden und deswegen mache ich Mathe jetzt für das GruMi-Lehramt.“

“I really want to work with people and I had difficulties in studying for the upper secondary level, especially because of mathematics and then I decided for the children and therefore now I am studying for teaching at the primary and lower secondary level.”

This corresponds with her emphasis on empathy, humanity, patience and diagnostic competence teachers should have. Teachers should be able to put themselves into the students' position and to

recognise their problems, to show the needed feeling and sympathy and they must be persistent in giving explanation.

> *„Viel Geduld, auf jeden Fall mathematisches Verständnis, aber eben auch dieses Einfühlsame, dass sie verstehen, wer verweigert, wer kann einfach nicht und dann ein bisschen auch das Gefühl dafür wer kann vielleicht mehr, wo kann man noch reizen. Und ansonsten auch viel Menschlichkeit, glaub ich.“*
> *„Much patience, in any case mathematical understanding, but also this sensitivity for they can understand who refuses, who really is not able, and then also the feeling for who might be able to do better, or where one can demand more. And besides that, also much of humanity, I think.”*

She emphasises the role of practical courses because such competences one cannot learn from theoretical education. Thus, to her practical courses and own experiences are much more important.

> *„ …die Geduld hab ich, glaube ich, eher bei meinen Nachhilfeschülern gelernt.“*
> *„Und die Eigenschaften kann man höchstens gesagt bekommen, was man haben sollte, aber die muss man wohl in den Praktika üben. Deswegen denke ich sind die auch schon wichtig.“*
> *“...patience, I think I have learnt more from giving private lessons.”*
> *“And the capacities one can only be told, which one should have, but one needs to practice them in practical courses. That is the reason, I think, they are so important.”*

Altogether, her understanding is strongly influenced by her experiences with private lessons she gave to students.

For her the education in pedagogical content knowledge in mathematics forms a relevant component of the teacher education. Before that lectures she used to teach following their own feelings, now mathematics didactics provides her with a valuable tool for a qualified planning of lessons:

> *„ …didaktisch, denke ich, hab ich einiges gelernt, also damit habe ich mich vor dem Studium auch gar nicht mit beschäftigt […] das habe ich bis jetzt nur aus dem Gefühl gemacht, jetzt hab ich dafür auch ein paar Regeln gekriegt.“*
>
> *„...didactically, I think I have learnt a lot, and before studying I have not been engaged with that […] that I have done by my own feeling, now have also received some rules for it.”*

She underlines that subject-matter pedagogical content knowledge is dependent on subject knowledge and cannot exist without it. That subject-related pedagogical content knowledge and general pedagogical knowledge make an important contribution to the development towards professionalism, she did not yet realise at the beginning of her studies. First she regarded those seminars just as gap fillers in her timetable between subject-related courses before she realised its benefit:

> *„Ja, auf jeden Fall sehr wichtig, weil ich festgestellt hab, dass ich vieles jetzt einfach nur so mitgenommen hab, weil es steht ja auf meinem Stundenplan. Und man dann erst im Nachhinein erkennt, okay, dafür könnt ich das jetzt noch gebrauchen…“*
>
> *„Yes, by all means, it is important because I have learned that many things I have just chosen because it is on my timetable. And that only afterwards one realises, okay, I can need it for that too.”*

To resume, it can be stated that in her opinion subject-based education is important especially regarding the own profession, because the pedagogical content knowledge in mathematics depends on solid mathematical knowledge. Furthermore, meanwhile pedagogical content knowledge in mathematics has gained greater importance because it became fundamentally useful for choosing and

justifying methods when planning lessons. In connection with the development of professionalism she gives a central role to practical experiences, especially concerning the development of personality traits and the communication with students.

(2) The case of Ben

In Ben's beliefs about mathematics as a subject or a scientific discipline, dynamic perceptions are dominant. For Ben mathematics starts from questions and problems of everyday life and our natural environment for which he strongly separates himself from a perception which values mathematics as an abstract and formalistic structure.

> *„Das hat was mit deinem Leben zu tun und Mathematik umgibt uns. Es geht hier gar nicht um Taylorreihen oder irgendetwas Abgefahrenes, sondern um ganz praktische einfache Sachen."*
> *„…Mathematik ist für mich keine abgehobene, theoretische Wissenschaft für abgehobene, theoretische Wissenschaftler, sondern etwas, was mit meinem Leben zu tun hat und zwar immer dann, wenn da Zahlen und die Verbindung von Zahlen oder Mengen eine Rolle spielen."*
> *„ It has something to do with your life and mathematics surrounds us. It is not about taylor series or spacy things but about purely practical and simple things."*
> *"… for me, mathematics is not a theoretical aloof science for theoretic aloof scientist, but something which has to do with life, and always has to do with life when numbers or the connections between numbers or amounts are playing a role."*

Relation to reality and everyday life is a very important part of his *beliefs* about mathematics. He understands mathematics as an instrument which helps him to handle everyday life problems better.

> *„Dann brauche ich unter Umständen mathematisches Handwerkszeug, um Sachen vergleichen zu können, zusammenfassen zu können, auch bewerten zu können, wenn ich irgendwo eine Statistik lese und mir da erzählt wird, da ist irgendwo ein Anstieg von zehn Prozent von irgendetwas, dann muss gerne wissen, was bedeutet das."*
> *„Then I possibly need a mathematical instrument which enables me to compare things, to resume, to evaluate, when somewhere I read a statistic that tells me that there is an increase of ten percent of something, then one will be happy if one knows what that means."*

Formal aspects only play a minor role for him: He underlines that formal proofs are also meaningful but they are much less meaningful than the solution of everyday problems by the application of mathematics.

> *„Ich habe selbst in meinem Matheunterricht […] viel mehr Techniken und Prozeduren gelernt, als inhaltlich verstanden, worum es geht. Ich habe erst hier an der Uni erfahren, wofür Analysis und Ableitungen da sind. […] Wozu ist das eigentlich gut, im richtigen Leben, in der Praxis. War mir nicht klar."*
> *„I myself have learned more techniques and procedures in my mathematics lessons […] than I have understood the contents of them and what they where about. I have learned not until the university what's the use of calculus and derivates. […] For what one can use that, in real life, in practice. That was not clear to me."*

For him, the real world context is an indispensable part of mathematics. He emphasises the modelling circle which plays a central role when dealing with real world examples:

> *„…einmal die Reduzierung der Daten, die mir die Realität vorgibt auf das mathematische Modell, da muss ich filtern und mich auf das, was ich wesentlich finde beschränken. Wenn ich da grobe Fehler mache, dann ist natürlich meine mathematische Lösung zwar aus dem mathematischen*

Modell richtig errechnet, aber sie sagt mir nichts oder wenig über die Realität, oder möglicherweise sagt sie mir etwas Falsches."
„on the one hand the reduction of the data, which was given to me on the mathematical model by reality, there I have to filter and concentrate on what I consider to be essential. If I make a gross error there, than of course my mathematical solution is indeed calculated correctly out of the mathematical model, but does not tell me anything or few about reality, or potentially tells me something wrong."

Within the modelling process he considers the phases of mathematisation and interpretation as especially important, the formal calculation is less important to him. For him the work within the mathematical model is a formalism, which cannot be executed without prior mathematisation und which is more or less worthless without reinterpretation. Again one can see clearly that for Ben using mathematics as a tool for solving extra-mathematical problems is more important than calculating mathematical models according to a fixed scheme.

Corresponding with this, one can find his statements about the significance of proving in mathematic lessons, which he considers to be important, as proving makes the generality of certain issues clear for the students. On the other hand he wants to choose the kind of proof highly in relation to the level of the learners:

„Wenn ich sehe, das Niveau der meisten Schüler ist so, dass sie alle mit dem präformalen Beweis glücklich sind, dass aber die meisten von denen den formalen Beweis noch vertagen könnten, dann sollen sie ihn auch gerne haben. Wenn ich das Gefühl hätte oder sogar die Gewissheit, dass ich bei 90 Prozent der Schüler, wenn ich jetzt den formalen Beweis mache, die Erkenntnis aus dem Präformalen wieder zerstöre, dann würde ich sagen lassen wir den."
„If I see, the level of most of the students is like they are all happy with the preformal proof, but most of them could do with the formal proof, then they can willingly have it. If I had the feeling or even the certainty, that I would destroy the cognition out of the preformal for 90 percent of the students, if I did the formal proof now, then I would say we leave it out."

His *beliefs* about teaching and learning mathematics correspond with his dynamic conceptions of mathematics as a process or activity. Thereby he describes the high algorithmic orientation and its focus on learning and executing schematic procedures as a problem of German mathematics lessons.

„…ich habe auch in der Hospitation mal einen Lehrer gesehen, der bei Dreisatzaufgaben immer schwer nach Schema gegangen ist […] ich würde viel mehr darauf drängen, einmal das Prinzip zu verstehen und sich eben inhaltlich klar zu machen, bei so einer Aufgabe [worum es geht]…"
„…once I saw a teacher during one of my school practicum, who always proceeded with proportional exercises strictly on the basis of a scheme […] I instead would insist much more on the one hand on understanding the principle and on making clear for oneself, such an exercise [what it is all about]…."

He in contrast highlights that it is important to show the practical use of mathematics for the students´ everyday living and by doing so to motivate the students and to take away their mathematics anxiety.

„Wichtig ist natürlich auch den Schülern die Angst zu nehmen […] auch mit dem Hinweis darauf, das kann nicht nur der Eine, der hier gut in Mathe ist, sondern das könnt ihr alle. Ihr müsst hier keine Angst vor haben, es beißt nicht und ihr könnt heut einfach nach Hause gehen und sagen, das habe ich gemacht, das weiß ich, wie das geht und da habe ich was verstanden."

> *"Of course it is also important to take away the anxiety from the students […] also with hint that not only the one who is good in maths here can do it, but all of you can do it. You don´t have to have any fear of it here, it doesn´t bite and you can simply go home today and say, I have done this, I know this, how it works and there I have understood something."*

Thereby practical, action-orientated aspects should be in the foreground also for proving, mathematics should be made concrete and descriptive, and that is why pre-formal proofs should play a decisive role in teaching mathematics. Though his concept of mathematics is a constructivist and students–orientated one, responding to the strengths and weaknesses of the learners and trying to produce references in their language and to tie up to their conceptions. Doing so for him mutual respect plays a central role, which means an engagement on the part of the teacher who takes the students serious and is really interested in them. These students–orientated conceptions of teaching mathematics are accompanied by a high reference to group working in teaching in which the teacher is more a moderator instead of giving the impulses and where the students clarify their understanding problems on their own first.

His beliefs towards teacher education and towards the professional development of future mathematics teachers are influenced by the experience of the distance between his mathematical education at the university and his practical experiences in school. In Ben's opinion his studies of mathematics at the university have prepared him meanly for the mathematics needed in school and in his opinion this part of his studies is more or less a necessary leg which one has to do for becoming a teacher but which has no use for his professional qualification. The motivation which is necessary to "struggle through" theses studies only results out of his practical experiences in school.

> *„Gerade Mathe an der Uni, unzählige Tage, an denen ich dachte ich bin hier falsch, ich bin zu doof, was auch immer, aber wenn ich damit an der Schule war, dachte ich, doch dafür lohnt es sich durch das Studium zu kommen."*
>
> *"Especially maths at the university, countless days when I thought I am wrong here, I am too stupid, whatever, but when I was at a school with this, I thought, however that is worth coming through the studies."*

In his opinion there is little use in the studies of mathematics for the later subject teaching, in contrary one has to feed on one's own school experiences.

> *„Das heißt, wenn mich jetzt jemand fragt, sagt mal schnell Geometrie neunte Klasse, kann ich mir das Buch nehmen und möglicherweise kann ich mir das auch relativ schnell zusammenreimen, aber zusammenreimen heißt aus eigener Schulerfahrung, nicht aus dem Studium."*
>
> *"That means, if someone asks me now, say geometry ninth grade quickly, I can take the book and maybe I can make sense of it relatively quickly, but making sense of it means by own school experiences not by the studies."*

Furthermore he stresses the necessity to be taught a broad knowledge basing on school mathematics, quasi a "sweeping statement" for mathematics, giving you a general overview and preparing you for your later work as a teacher. He attributes a high significance to the didactical education for the professional development and thinks that problems in teaching mathematics at school often result out of a lack in knowledge related to this area.

> *„…ich muss keine hohe Mathematik machen, sonder wirklich relativ einfache Geschichten. Das höchste der Kunst ist, dass ich in der höheren Handelsschule mal ganz bisschen Kurvendiskussion*

und ganz bisschen Matrizenrechnung mache. […] Und da sehe ich wirklich die Herausforderung mehr in der Didaktik…"
"… I don't have to do high mathematics, but truly relatively simple stories. The highest of the art is that I sometimes do very little curve sketching and very little matrices arithmetic at the commercial college. […] Und there I really see the challenge more in the didactics."

He stresses the necessity of a higher significance of the pedagogical content knowledge in mathematics which has a too low value especially in contrast to the mathematical studies which are provided by the university with a higher amount of contact hours per semester and a higher acceptation. Hence in his opinion the didactics are often completed secondarily and sometimes are used as "gap fillers".

„Man hat im Grundstudium Sorge vor der Zwischenprüfung Analysis vor allem. Aber, wenn man die gemacht hat, da guckt man wann im Stundenplan Platz ist und klatscht dann irgendwie diese Fachdidaktik noch mit rein und das ist dann schnell erledigt, macht keinen Stress. Man muss halt vielleicht da sein, aber so nach dem Motto: Da kann man auch gar nicht durchfallen…"
"One worries about the intermediate examination, calculus mainly during the basic studies. But, when you have finished it, then you look when there is space in the schedule and then you slap these subject didactics also in it somehow and that is done quickly then, cause no stress. Maybe you have to be there, but themed like: There you can´t fail."

Besides the didactical training he also emphasizes the high value of personal characteristics and a general interest in children and the work with them as important for a profession as a teacher.

„…die Liebe zur Person, zum Menschen, dass ist etwas, was den Pädagogen auszeichnet. Ich muss ein Interesse daran haben, an den Schülern, mit denen ich da zusammen arbeite. Das sind nicht Schüler Nr. 1 bis 25, sondern die haben Namen, die haben alle eine eigene Biographie."
"… the love towards a person, towards human beings, that is something, what is characterizing the pedagogue. I have to have an interest on it, on the students, with whom I am working with. These are not my students no. 1 to 25, but they have names, the all have there own biography."

In summary his statements about the education at the university clearly show that for him as a future teacher at a vocational school especially the education in mathematics as a science has little value for his later work. He points out that he has not been able yet to use mathematical contents learned at the university for his own teaching but always had to go back to the knowledge of his own schooldays during his practical phases. He stresses the necessity of a higher importance of lectures in pedagogical content knowledge in mathematics which in his opinion are taught in a much too small extend. On the other hand in his opinion one will acquire a big amount of one's professionalism during the second and third phase of the teacher education and during one's own teaching, provided that one has the required personal characteristics for becoming a teacher which can not be taught by any training's activities.

4. Summary and conclusions

The results of the analyses of the interviews about *mathematical beliefs* can be summarized on the following three levels

- Concerning the *beliefs* towards mathematics as a subject one can find the biggest differences between Anna as a student in her third semester and Ben as a student being shortly confronted with his examination. Anna shows a more or less static view on mathematics whereas dynamical conceptions clearly dominate in Ben's opinion.
- With regard to the *beliefs* towards teaching and learning Ben's dynamical view continues in constructivist concepts about teaching and learning mathematics. Ben understands mathematical lessons as an active process starting with questions and problems resulting from everyday life.

One the one hand similar *beliefs* can be reconstructed from Anna's statements. But on the other hand she also often shows beliefs emphasising the more receptive side of teaching and learning mathematics. This corresponds with her more static view on mathematics.

- Concerning the professional development of teachers both Anna and Ben stress the high importance of practical experiences and subject based didactical education.

In detail:
With regard to the *beliefs* toward mathematics as a subject a more statistic view on mathematics, that is a view basing on mathematical formalisms, appears in Anne's answers as she is just at the beginning of her studies. This view could nearly be expected because Anne is a beginner in university study and starts her studies influenced by her own experiences of mathematics lessons in school. Because of these experiences she tends to traditional opinions towards teaching and learning mathematics, a fact which is indicated on in literature time and again (cf. Ball, Lubienski und Mewborn, 2001). Her beliefs are impressed by a static tendency because obviously her own mathematics lessons were mostly algorithmic and schematic orientated, less dynamical and related to reality.

Ben shortly confronted with his examination disposes of several years of practical and professional experience. With him a clear predominance of dynamical *beliefs* towards the subject mathematics appears, which means more process- and application-orientated beliefs focussing more on an everyday life relation and less on executing algorithm. This is in accord with the state of the discussion about changes of attitudes during one' studies which point out that students change from conservative ideas of education at the beginning of their studies to more liberal attitudes at the end of their studies (cf. Dann et al. 1978). Furthermore this positive influence of the studies on the development of the *beliefs* can also be observed with Anne already to some extent, as she not only has a traditional formal view on mathematics, but also liberal attitudes towards mathematics can be reconstructed.

For both, Anna and Ben, their respective *beliefs* towards teaching and learning mathematics are strongly influenced by their previous experiences. Thus a tightened attitude towards mathematics and its educational value, which is also reflected in his *beliefs* towards the way how teaching works, appears with Ben at the end of his studies. His experiences from his previous completed vocational training and from the following studies of mathematics obviously have strongly influenced his *beliefs* towards teaching and learning mathematics. So one can see in his statements the closeness to constructivist concepts of practicing teachers as pointed out by the research done by Leuchter et al (2006) on the comparison of German and Swiss teachers. For example he phrases decidedly that mathematical lessons have to tie up to students' life and to their concepts to be successful. Mathematical lessons should be fewer teachers centred but more focussing on the active doing of mathematics and on active learning processes of the students. Furthermore one can reconstruct dynamical, application-orientated beliefs towards mathematical lessons, which shows that for him the extra mathematical use and the understanding of the contents are the focal points of mathematical lessons in contrary to a algorithm orientated use of techniques and procedures.

One can clearly identify the „filter function" of beliefs in Ben's ideas. Pajares (1992) describes this function as follows:
"Thought processes may well be precursors to and creators of beliefs, but the filtering effect of belief structures ultimately screens, redefines, distorts, or reshapes subsequent thinking and information processing" (p. 325).
Thus Ben's view on mathematics influences one the one hand strongly his concepts about teaching and learning mathematics, and on the other hand his ideas about the knowledge necessary for his future professional work.

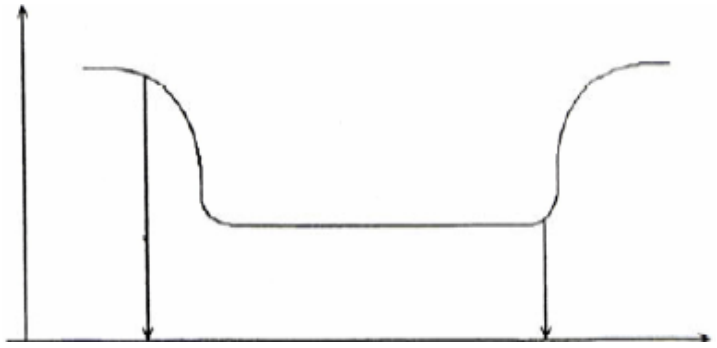
In contrary more still "swinging" *beliefs* towards teaching and learning mathematics can be reconstructed with Anne as a beginner with her studies. She partly has liberal conceptions about teaching, such as the use of open teaching methods or the avoidance of compulsion towards students. But on the other hand her *beliefs* towards teaching and learning often are still conservatively influenced describing the teacher as a pure transmitter of knowledge. Also her rather schematic conception of mathematics, with training schematic algorithm being in the foreground, corresponds with these rather conservative ideas. Here Anna obviously is strongly influenced by her own school days, and she has not yet gained enough experiences from practical courses in school which could have offered her different perspectives. Contrariwise Anna sometimes confines herself consciously from her own school experiences, which she - to some extent - does not want to adopt into her own teaching practice. She expects from her entire studies to learn ways to do it "better" in the future.

Relating to the *beliefs* towards education of mathematics teachers and their professional development, both Anne and Ben see a positive influence of the studies – or at least parts of them - on the professional development of mathematics teachers. Anna and Ben attribute a high importance to the pedagogical part of their studies and there especially to the pedagogical content knowledge in mathematics. Both describe that they could sensibly capitalise on their courses in mathematics didactics, because there they were given concrete tools for planning lessons, which they only had developed on a gut level before. Especially Ben highlights that the didactics are neglected at the university und do not take the position which is entitled to them. He attributes a significant role for teachers' professionalizing to education in pedagogical content knowledge in mathematics.

These corresponding statements indicate on the big potential of didactical courses in view of changing beliefs, as these didactical courses have a direct relation to the further professional field und hence can directly take up with the beliefs related to mathematical teaching. However, both Anne and Ben regard the practical experiences, which will be made during and after their university studies as the most important part of teacher education and teachers professionalizing. These self conducted lessons and furthermore the practical work with children and students allow an active learning process in which beliefs and cognition can be gained or changed by own experiences.

In contrast differences concerning the valuation of the studies appear in relation to the time one has already been studying. Anna as a beginner still has high expectations towards her teacher education at university. She expects a big benefit for her own professional development from the entire university studies, which means also from the pure mathematical education. Ben as a student in the end of his studies is considerable more "disenchanted". He sees little use for his later practice in school especially in the knowledge taught in mathematics. For him teacher education at the university fails the needs of the later demands in practice and so does not help with regard to the professionalizing of becoming a teacher.

Contrasting both students and their beliefs in general one can reconstruct the first part of the so called „Bowl of Konstanz" ("Konstanzer Wanne") as described in literature. According to the research of a research group from Konstanz about the change of attitudes of future teachers, these future teachers have quite "conservative" beliefs at the beginning of their studies which decline during their studies in favour of more "progressive" beliefs. After first contacts with real work in school practice these progressive beliefs are – in consequence of the so called practical shock – substituted again by more conservative beliefs being geared to conformism with the teaching staff and tending to more compulsion towards the students (cf. Dann et al. 1978). This so called "Bowl of Konstanz" can be demonstrated roughly as follows (the arrows mark the beginning of the studies and the practical shock):



Although these descriptions are not without controversy, they show a clear closeness to the Double Discontinuity described at the beginning by Felix Klein, who traces this development to subject reasons. Precisely the description of the practical shock and as a consequence of it the return to traditional teaching methods, according to Felix Klein, is also released by the circumstance that the university has not provided the knowledge necessary for practical work in school. This criticism is formulated exactly in this way by Ben who is very disappointed by his studies, especially by the education in mathematics and who wishes more knowledge focusing on school knowledge. This demand correspond with the conceptualisation of pedagogical content knowledge developed in PTEDS and as it is described in the beginning (cf. Tatto et al. 2007, Blömeke, Kaiser, Lehmann, 2007). This demand upon teachers' professional knowledge can also be found in the work of the COACTIV-project (cf. Krauss et al., in print) and in the work of Ball, Hill, Bass (2005) who claim that mathematics teachers should have mathematical knowledge being composed of the following two components: „'common' knowledge of mathematics that any well-educated adult should have and mathematical knowledge that is ‚specialized' to the work of teaching and that only teachers need know." (p. 22)

Due to the case study character of this description it is neither intended nor possible to generalise the results of these interviews. It has especially to be considered that the case study is based on a convenience sample and not a representative sample, although these two cases are based on the interviews with more students, who describe similar aspects. However, the description of the cases of Anne and Ben shows exemplarily structural weaknesses of teacher education to which Felix Klein already referred at the beginning of the twentieth century, and whose adjustment is not fulfilled. The critics is still topical, such as the remoteness from reality of teacher education, which is often complained about, the too little interlocking with practical parts of the studies, the missing relations between the taught mathematical contents and the knowledge needed in school, and the too little significance of didactical courses which could in the sense of Bromme (1994, 1995) build a bridge between the different components of teacher education.

Günter Törner has worked for the change of teacher education for future mathematics teachers in many initiatives which aimed and still aim for overbearing the Double Discontinuity in the sense of Felix Klein by a well understood subject education in mathematics – that is elementary mathematics from a higher perspective. Thus, Felix Klein describes in the foreword of his "as subsuming lectures" conceived courses as its aim:

„Meine Aufgabe hier wird stets sein, Ihnen den *gegenseitigen Zusammenhang der Fragen der Einzeldisziplinen* vorzuführen, der in den Spezialvorlesungen nicht immer genügend zur Geltung kommt, sowie insbesondere ihre *Beziehungen zu den Fragen der Schulmathematik* zu betonen. Dadurch, so hoffe ich, wird es Ihnen erleichtert werden, sich diejenige Fähigkeit anzueignen, die ich

doch als eigentliches Ziel Ihres akademischen Studiums bezeichnen möchte: *daß Sie dem großen Wissensstoff, der Ihnen hier zukommt, einst in reichem Maße lebendige Anregungen für Ihren eigenen Unterricht entnehmen können*."

"My task here will always be, to demonstrate to you the *mutual relation between the questions of the individual disciplines*, which not always shows to advantage in the specialised lectures sufficiently, furthermore especially to stress their *relations to questions of school mathematics*. Thereby, so I hope, will it be more easy for you, to acquire that ability, which I would like to refer as the actual aim of your studies: *that you once can take agile suggestions for you own teaching on a high degree from the vast amount of topics, which comes up to you here*."

From: Felix Klein (1933), Elementary mathematics from a higher perspective. Vol. 1 (Elementarmathematik vom höheren Standpunkte aus. Erster Band.) p. 2.

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