

COMPLEX SYSTEMS IN THE ELEMENTARY AND MIDDLE SCHOOL MATHEMATICS CURRICULUM: A FOCUS ON MODELING

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Abstract: *An appreciation and understanding of the world as comprising interrelated complex systems is critical for all citizens in making effective decisions about their lives, their families, their communities, and the environment. This paper argues for the need to introduce elementary and middle school children to the rudiments of “basic complex” systems within the mathematics curriculum. A powerful avenue for doing so is through mathematical modeling. Following an overview of the key features of complex systems and mathematical modeling, a sample problem is presented to illustrate one way in which children can engage with basic complex systems. Some ways in which 4th and 5th grade children solved the problem are presented.*

Introduction

Studies of complex systems have proliferated the literature in the past decade (e.g., Bar-Yam, 2004; Hmelo-Silver, Marathe, & Liu, 2007; Jacobson, 2001; Jacobson & Wilensky, 2006; Lesh, 2006). Calls for school curricula to increase students’ exposure to complex systems are clearly warranted, given that we live in a world that is increasingly governed by such systems: the World Wide Web, political parties, financial corporations, school and university systems, family structures, and sporting teams are just a few examples. An appreciation and understanding of the world as comprising interlocked complex systems is critical for all citizens in making effective decisions about their lives as both individuals and as community members (Bar-Yam, 2004; Jacobson & Wilensky, 2006; Lesh, 2006).

Research addressing students’ understanding of complex systems has been confined largely to the secondary and undergraduate years, with the domain of science being a popular field of investigation (e.g., see Jacobson & Wilensky’s review, 2006). Studies of complex systems have had virtually no impact on mathematics education (English, 2007a). This is not because mathematics educators have been ignorant of such developments; rather, current learning science theories do not provide sufficient answers to how we can promote students’ understanding of complex, real-world systems that permeate their life. As Lesh (2006) noted in the complex systems strand of the *Journal of the Learning Sciences* (15, 1):

I do not view current learning science theories as being sufficient to provide answers to most questions about the nature of the conceptual systems that students would need to develop to understand complex

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systems. In contrast, I believe that the most exciting point about learning science investigations of complex systems is precisely that such research is likely to require a variety of significant paradigm shifts beyond current ways of thinking. Furthermore, I believe that these paradigm shifts should have implications for learning and problem solving related to a wide range of constructs and situations where relationships to systemic understandings are far less obvious than in the case of complex systems (p. 46).

In this paper, I add further support to Lesh's (2006) argument that current complex systems research is not sufficient for addressing the nature of the conceptual systems that students need in dealing with "everyday" complexity in their world. What this research is providing, however, are important theoretical shifts in how we can address complex problem solving and learning in the mathematics curriculum. I present here an alternative perspective on developing students' facility with complex systems. My focus is not on the "deeply complex" systems that involve many layers of complexity where the "agents" within the system are often living organisms or ecosystems (Lesh, 2006, p. 47). Rather, I consider "basic complex" systems that are inherent in authentic mathematical modeling problems designed for elementary and middle school students. In order to appreciate the nature of these systems, I first review briefly some of the key features of deeply complex systems. I then identify the main components of basic complex systems, which I have been introducing to elementary and middle school students through mathematical modeling (e.g., English, 2007b). Following an overview of modeling, I provide an example of a problem involving a basic complex system and present some examples of how fourth- and fifth-grade children worked the problem.

"Deeply Complex" Systems

Understanding complex systems is fundamental to understanding science. The complexity of such systems makes them very difficult to understand because they are composed of multiple interrelated levels that interact in dynamic ways (Hmelo-Silver et al., 2007, p. 307).

As previously noted, studies that have explored students' understanding of complex systems within the biological sciences have been popular (e.g., Hmelo-Silver et al., 2007; Hmelo-Silver, Holton, & Kolodner, 2000). This is not surprising, given the many instances of deep complexity that abound in this domain. For example, processes that take place within organisms, the interactions between organisms and their environment, and the interactions among various organisms within and across species are addressed in terms of complex systems (Hmelo-Silver et al., 2007). Although such studies have some import for mathematics education, as I indicate here, they nevertheless address a more advanced level of complexity than is warranted in the elementary and middle school mathematics curriculum.

Defining complex systems per se has been a challenge in itself. Over a decade ago Horgan (1995) published an article titled, *From Complexity to Perplexity*, in which he highlighted the struggle of researchers to find a unified definition. One basic definition is that complex systems comprise sets of interconnected elements (parts) whose collective behaviour arises in an often counterintuitive and surprising way from the properties of the elements and their interconnections (Jakobsson & Working Group 1 Collaborators, http://necsi.org/events/cxedk16/cxedk16_1.html). For example, if we consider a complex system such as proteins, we have their elements (e.g., amino acids), their interactions (e.g., bonds), their formation (e.g., protein folding), and their activity (e.g., enzymatic activity; Bar-Yam, 1997). The properties, however, only become meaningful within the context of a system functioning as-a-whole; in other words, the whole is more than the sum of its parts (Lesh, 2006). At the same time, we cannot just focus on the whole; we need to move back and forth between seeing the parts and the whole. That way, we can see which aspects of the parts are relevant to describing the whole. This relationship between the parts and the whole is referred to as *emergence* (Bar-Yam, 2004).

Another key feature of the relationships between parts and wholes is *interdependence*, namely, a consideration of how the parts of a system affect one another (Bar-Yam, 2004). Here the focus is on the strength of the dependencies between the parts, and how manipulating or removing a particular part will

affect the rest of the system. Sometimes the effect is small and at other times, large, and there might be many effects on the system or only a few. These different kinds of interdependence are a major consideration in our efforts to solve a variety of problems, ranging from family issues through to major global concerns. It is thus important that students be given opportunities to investigate these different interdependencies in their efforts towards understanding relatedness and relationships. The challenge for the mathematics education curriculum is how to integrate experiences involving complex systems that introduce students to some of these fundamental features. One approach I suggest here is through experiences with basic complex systems (or “simply complex” using Lesh’s term, 2006, p. 47), which deal with meaningful real-world situations that engage students in generating significant mathematical constructs and processes.

“Basic Complex” Systems

Most of the complex systems that students encounter in their daily lives are really only complex at the highest level, with their component parts following rather simple functional rules (Lesh, 2006). This is in contrast to the deeply complex systems found in the biological sciences, where additional layers of complexity tend to emerge because the living organisms within the system are not generally governed by simple rules. Despite these differences, the two systems share core features. Those features that pertain to the basic complex systems I address here include:

- Sets of interconnected parts
- Interactions among the parts
- A system functioning as-a-whole
- Property of emergence: relationship between the parts and the whole
- Property of interdependence: how the parts of the system affect one another.

To introduce children to basic complex systems, I have been implementing mathematical modeling problems in elementary and middle schools (e.g., Doerr & English, 2003; English, 2007b; English, 2006; English & Watters, 2005). These problems, as I indicate later, normally display the above features and provide students with experiences in dealing with “everyday” complex systems. Prior to addressing an example of such experiences, I provide some background information on mathematical modeling for the elementary and middle school.

Mathematical Modeling for the Elementary and Middle School

Mathematical modeling is foundational to modern scientific research, such as biotechnology, aeronautical engineering, and informatics. Because mathematical models entail a focus on the structural characteristics of phenomena (e.g. patterns, interactions, and relationships among elements) rather than surface features (e.g. physical or artistic attributes), they are powerful tools in predicting the behaviour of complex systems (Harel & Lesh, 2003).

The terms, *models* and *modelling*, have been used variously in the literature, including with reference to solving word problems, conducting mathematical simulations, creating representations of problem situations (including constructing explanations of natural phenomena), and creating internal, psychological representations while solving a particular problem (e.g., Doerr & Tripp, 1999; English & Halford, 1995; Gravemeijer, 1999; Greer, 1997; Lesh & Doerr, 2003; Romberg et al., 2005; Van den Heuvel-Panhuizen, 2003). The perspective that I have adopted in my research is that of models as “systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behavior of some other familiar system” (Doerr & English, 2003, p.112).

Mathematical modeling takes children beyond the usual form of problem solving they meet in the elementary school. Typical classroom problems present the key mathematical ideas “up front” and children select an appropriate solution strategy to produce a single, usually brief, response. In contrast, modeling problems have

the important mathematical constructs and relationships embedded within the problem context and children elicit these as they work the problem. The problems necessitate the use of important, yet underrepresented, mathematical processes such as constructing, describing, explaining, predicting, and representing, together with organizing, coordinating, quantifying, and transforming data—all of which are important for dealing with complex systems in our world (e.g., Gainsburg, 2006; Lesh & Doerr, 2003). The problems are also multifaceted and multidisciplinary: students' final products encompass a variety of representational formats including written text, graphs, tables, diagrams, spreadsheets, and oral reports; the problems also cut across several disciplines including science, history, environmental studies, literature, and history.

Importantly, these modelling problems encourage the development of generalizable models. In my research with elementary and middle school children, I have implemented sequences of modeling problems that encourage the creation of models that are applicable to a range of related situations (e.g., Doerr & English, 2003; Doerr & English, 2006; English & Watters, 2005). Children are initially presented with a problem that confronts them with the need to develop a model to describe, explain, or predict the behavior of a given system (a model-eliciting problem). Given that re-using and generalizing models are central activities in a modelling approach to learning mathematics and science, the children then work related problems that enable them to extend, explore, and refine those constructs developed in the initial problem (model-exploration and model-application problems; Lesh, Cramer, Post, & Zawojewski, 2003). Because the children's final products embody the factors, relationships, and operations that they considered important, powerful insights can be gained into the children's mathematical thinking as they work the problem sequence.

Finally, unlike typical school problems, modeling activities are social experiences (Zawojewski, Lesh, & English, 2003), where children engage in small-group collaborative work and are motivated to challenge one another's thinking and to explain and justify their ideas and actions. Numerous questions, issues, conflicts, revisions, and resolutions arise as children develop, assess, and prepare to communicate their solutions. The ability to work collaboratively on comprehensive problems in which planning, monitoring, and communicating results are essential to success is not normally developed in typical classroom problem solving; dealing effectively with complex systems calls for such skills.

An Example of a Model-Eliciting Problem Involving a Basic Complex System

"Christmas Dinner at the Thompson's" (Appendix A) is a model-eliciting problem that I presented to 55 fourth- and fifth-grade students (9- and 10-year-olds) who attended a two-day "Mathematical Modeling Challenge" for schools in south-eastern Queensland, Australia. The children worked the problems in groups of three to four during the second day of the Challenge activities (3 sessions totalling approx. 3 hours were devoted to the Dinner problem). After the children had solved the problem, they presented group reports on their models and their creations to the class. The reporting session was an important aspect of the problem activity as it allowed the children to compare and contrast ideas, to provide constructive feedback to their peers, and to build on ideas to create new understandings (cf. notion of knowledge building, Bereiter & Scardamalia, 2006).

As the children worked the problem, the research assistants and I moved around the classroom observing the children's interactions and, where appropriate, asked them to explain or justify their ideas. No direct teaching was presented; if the children sought assistance we would advise them to seek answers within their group and, if necessary, would pose one or two questions to refocus their thinking. We video-taped and audio-taped both the children's conversations and actions, and their final class presentations.

The Christmas Dinner problem comprises several components: background information (in text and table format) that sets the problem context and alerts the students to some factors they need to consider in working a problem of this nature (Part A); "readiness questions" (not included in Appendix A) designed to ensure the students have understood the information presented; additional data and information for consideration in working the problem (part B); and the problem "challenge" itself (Part C).

As can be seen, the problem is multifaceted and comprises (a) sets of interconnected parts (e.g., sets of various ingredients; food items to be oven baked; components of cleaning and decorating; responsibilities and tasks to be shared among the children) and (b) interactions among these parts (e.g., how and when the food

items will be placed in the oven and what will be placed with what [and who will oversee this]; how cleaning and food preparation will be integrated; how the available bench space will govern food preparation). The overall system (schedule) must function as a whole for the dinner to be a success. For example, the three children need to coordinate all of their actions efficiently, taking into consideration the various tasks that have to be done and when they have to be done, the availability of needed utensils and bench space, and the timing of each task. As the children develop and implement their system, they need to assess how the various parts are working towards creating the whole (the dinner) [emergence property]. They also need to consider how the various parts affect one another (interdependence). For example, in preparing their parents' anniversary dinner (Part A), the children overlooked how the availability of kitchen equipment affected the timing of the different tasks.

How Children dealt with the Basic Complex System in the Dinner Problem

In this section I present a few group reports that the children presented to their class peers on completion of the Dinner problem. Their reports indicated the children's awareness of the multifaceted nature of the problem, with the need to consider each of the different parts, their interactions and interdependence, and their functions in forming the overall system.

The "Marvellous Mathematicians" group explained their reasoning in creating their schedule as follows. The actual schedule they developed appears in Appendix B. This activity is all about organisation and planning. The challenge we are faced with is to organise a Christmas Dinner schedule for the Thompson family kids. They are doing this for their hard-working parents to make their limited holiday time relaxing and enjoyable. They have made a dinner before but they made a mistake in the planning and all, and all needed to use the oven at the same time. In our schedule we need to include, who uses what, when and for how long. We also need to include smaller chores and cleaning time. When asked for further explanation of the development of their schedule, the children stated:

We started by figuring out what time we wanted to serve each different dish. Then from that information we could figure out what time Dan, Eva, and Sophie needed to start cooking the different dishes so that they would be ready in time. Next we made a schedule showing who cooked what and when. We also made an extras chart showing who did the cleaning, decorations and other small tasks. I followed up by asking the group to identify some of the aspects of the problem that presented a challenge to them. They explained:

Well sometimes we kept getting pressed for time; they needed to do two things at the same time so we had to fix that... Um, like we just made them do all the activities and in the end it did work out...sometimes [we] had to switch a few things round...cause sometimes there would have been like half an hour when they weren't doing anything so we moved everything down a bit of time

Another group, the "Magnificent Mathematics Monkeys" stressed the importance of developing an effective schedule and explained how they followed a pattern in achieving this:

Dan, Sophie and Eva have decided to make a delicious dinner for their hard-working parents. The most important part of the dinner is the schedule. Without it the night would be a wreck. Our task as a group was to create a schedule that if followed would have the night run smoothly. Our group started off with the timetable for the most obvious facts and then we proceeded to add information where it was needed. As we progressed with the project we started to get into a pattern. We would do two activities and then we would clean up in the kitchen. Using this pattern we um, this pattern and other solutions we successfully finished the project.

When asked to explain their "pattern," the children responded:

Um, well when we started working, instead of doing one task and then cleaning up afterwards we found that we saved more time doing two tasks and then cleaning up after that. So we were saving time when we did that.

A third group, the “Banana Splits” highlighted the elements they took into consideration in developing their schedule and also noted that their system would be applicable to other related problem situations. They explained their procedures as follows:

How we came up with the schedule. To make the schedule we considered the time it takes to cook it, what utensils to cook each thing, but we also took into consideration whether they can cook anything else and whether each thing is warm or cold. With all these elements and a bit of trial and error we came up with our solution. This schedule is reusable for other events because of the list of elements above. Enjoy!

On asking the group to elaborate on the important components that made their system reusable, the children explained:

How long it takes to cook it, what um, dishes were needed to cook each thing, like what utensils, and we also took into consideration whether they could cook with anything else at the same time, and yeah, and if they were served warm or cold.

On conclusion of the group reports, we asked the children to reflect on the factors that they considered important in developing their schedule. The following discussion took place:

Andy: Time as in how long it takes to cook, how long it takes to prepare, how long everything’s going to take and how much time you actually have...

We asked if time was “one of the first things you needed to consider?” Lachlan responded:

Yes you have to consider how much time you have to cook everything cause if you don’t get it in at the right time, its going to be like, say main course, entrée, dessert, something like that!

Other children commented that one also needs to know: (a) “When things are going to be served,” (b) that “You have to use all three children,” (c) that “You have to know what utensils and things you need and how much space you have or how much oven space you have,” and (d) “You’ve got to make sure you have the ingredients necessary, not like go to make it and expect you have all the ingredients and go out and buy them.”

In a follow-up discussion on other situations in which they could apply their model, the children commented that they could apply it to other occasions such as birthdays, Easter lunches, and anniversaries. In extending their thinking here, we asked for some different situations. James responded:

Planning for a competition, like a touch [football] competition, it’s not quite the same but you’re still planning it, like what time that event is on, like that team and that team are versing [opposing] each other at that time...and how long the competition would go and then like quarter finals, semi-finals...

Another child suggested that their model could be applied to “bus and train timetables, with a schedule for when trains and buses are going to leave...”

Discussion and Concluding Points

This paper has argued for the inclusion of experiences with complex systems in the elementary and middle school mathematics curriculum. Although there has been substantial research on students’ understanding of

complex systems, especially in the biological sciences, such research has had almost no impact on the mathematics curriculum. However, studies of complexity are providing us with new ways in which we can structure problem-solving experiences that expose children to everyday complex systems. I have proposed here the notion of “basic complex” systems as a way of introducing children to examples of complexity in their world. This notion differs from “deeply complex” systems that involve many layers of complexity and usually involve living organisms or ecosystems.

Basic complex systems have formed the basis of the mathematical modeling problems I have introduced to elementary and middle school students. These systems share many of the features of the deeply complex systems: they comprise sets of interconnected parts with interactions among the parts; the systems function as a whole; and they display the properties of emergence (relationships between the parts and the whole) and interdependence (how the parts of the system affect one another).

The modeling problems involving basic complex systems that I have designed and implemented in elementary and middle school classrooms extend children’s problem-solving experiences beyond their usual curriculum. Rather than being presented with “neatly packaged” mathematics, as in standard word problems, children have to elicit the important mathematical ideas and processes themselves as they work the problems. That is, the problems require children to make sense of the situation so that they can mathematize it themselves in ways that are meaningful to them.

In the Christmas Dinner problem presented here, children dealt with some of the key features of basic complex systems as they developed a schedule for organizing a family function. The children’s class presentations and written artefacts displayed their awareness of the comprehensive nature of the problem and the need to consider each of the different parts, their interactions and interdependence, and their functions in creating the system itself. The children could also identify other situations in which they could apply their models.

Mathematical modeling provides a rich avenue through which children can learn about basic complex systems. Because they regularly encounter such systems in their everyday lives, children need more exposure to mathematical problems that equip them with skills to work with complex systems. The inclusion of such problems in the regular mathematics curriculum is paramount. The level of complexity children experience in their world is increasing rapidly—we need to ensure they can deal effectively with this complexity.

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AUTHOR NOTE: Acknowledgements

The data reported here are from a study funded by a grant from the Australian Research Council. Any claims in this article are mine and do not necessarily represent the position of the Council. The support given by my research assistant, Jo Macri, is gratefully acknowledged.

PART A
Christmas Dinner at the Thompson's

John and Jan Thompson own and operate a convenience food store called the 'Food Owl'. Their shop is open seven days a week from 7 am to 7 pm. Both John and Jan work long hours in the store and very rarely get a day off.

Christmas is fast approaching and John and Jan Thompson are even busier than normal. People are coming into their shop to stock up on Christmas foods, decorations, small gifts, and holiday treats. Whilst other families are busy preparing for the festive season the Thompson's are busy WORKING.



The Thompson's children, Dan, Sophie and Eva want to do something special for their parents this Christmas. They would like to make their parents limited holiday time as relaxing and enjoyable as they can.

Last year the children cooked a special dinner for their parents' Wedding Anniversary and they have decided to plan another celebration. This time they want to decorate the house, prepare, cook, and serve Christmas dinner for their Mum and Dad.

From their past experience Dan, Sophie and Eva know that planning a dinner party requires organisation and a well-planned schedule. Sophie said, "Once we do our schedule, the most overwhelming chore is done. Decorating, preparing and cooking the dinner are easy, but if the schedule isn't correct, then the whole dinner will never be ready!"

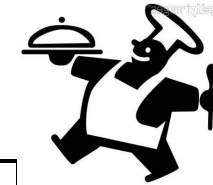
When the children organised their parents' Wedding Anniversary dinner they created an activity list and a schedule for preparing and cooking the meal.

Some of their jobs included:

Activity List



Menu and Instructions	Other Tasks
Chicken and sauce Preparation – chicken <i>30 min</i> Preparation – sauce <i>20 min</i> Cooking time <i>1 hour</i>	Set table <i>15 min</i> Organise serving dishes and utensils <i>15 min</i>
Potatoes Wash, dry and cut up <i>15 min</i> Bake <i>50 min</i>	Clean up <i>10-15 min</i> After each cooking part Dishing up and serving <i>15 min</i>
Salad and dressing Wash and cut all ingredients and make dressing. Refrigerate until dinner time <i>20 min</i>	Final clean up After serving <i>40 min</i>
Rolls Preparation <i>5 min</i> Warming time <i>10 min</i>	
Cheesecake dessert Preparation <i>30 min</i> Cooking time <i>1 hour</i>	



Preparation and Cooking Schedule

Dan	Prepare the chicken		Clean up from chicken	Prepare cream sauce for chicken	Clean up kitchen	Watch stove and oven		Dish food up and serve
Sophie	Make the dessert		Clean up from dessert	Wash salad, make dressing	Clean up from salad	Toss salad		
Eva	Wash potatoes	Set table	Whip cream for dessert	Clean up kitchen	Prepare serving dishes	Get rolls ready		

Unfortunately, the schedule they made did not work well. They found that they stumbled around the kitchen wanting to use the same equipment at the same time. For example, Dan wanted to cook the chicken at the same time both Sophie and Eva wanted to use the oven!

The Thompson children realised that they didn't think of all the things they needed to include in their schedule.

PART B
What's Cooking at the Thompson's?

The children have already decided the menu for their Christmas Dinner:



- Before dinner nibblies (cheese, dip carrot sticks and water crackers)
- Baked turkey, roast vegetables and steamed vegetables
- Pavlova, ice-cream and fresh strawberries

Dan, Sophie, and Eva know that their parents will be home from work on Christmas Eve at 6:30 pm. All the children are available to begin preparing the Christmas celebration at 2 pm. They have four and one half hours to get everything ready! The three children hope that the Christmas celebration they plan for their parents will be successful and a wonderful surprise for their hardworking parents. All they have to do now is organise a schedule that works better than the wedding anniversary schedule.

Here are some of the things they need to consider:

- how long the turkey will take to cook
- what other items can cook in the oven with the turkey
- when to decorate and set the table
- when to make the pavlova and how long it will take
- how often they need to clean in between the cooking
- how much bench space they have for food preparation
- what food needs to be ready first
- who will use the equipment and when



They also need to consider who will be responsible for what jobs!

Christmas Dinner



Items for dinner	Preparation time and instructions	Cooking time
Turkey	20 min to stuff and prepare	3 hours oven time
Stuffing	3 min – to be done first	Cooks with the turkey
Carrots, beans and broccoli	Wash and cut - 15 min	15 min steaming on the stove top
Roast potatoes and pumpkin	Wash and cut – 15 min	Roast in oven – 45 min
Salad	Wash and cut all ingredients – 15 min Make dressing - 10 min Toss salad – 5 min	Chill in fridge for approx. 30 min
Gravy	Mixing – 5 min	Heating on stove top – 10 min
Bread	Cut and put on baking tray – 5 min	Warm in oven – 10 min
Pavlova	Separate eggs and whip egg whites – 25 min	Bake pavlova in oven – 1 hour
Decorate pavlova	Whip cream – 10 min Wash and cut strawberries – 10 min	Prepare pavlova for serving – 10 min
Items to be ready at 6:30 pm		
Punch	Cut fruit mix juice and soft drink – 10 min	Chill for approx. 2 hours
Cheese and crackers	Cut cheese and put out crackers – 10 min	Do this as guests arrive home
Vegetables for dip platter	Wash and cut carrots into sticks – 10 min	
Dip	Mix all ingredients – 10 min	Chill for 1 hour

Other tasks which need to be completed

Task	Time
Clean house ready for celebration	30 min
Decorate house	45 min
Iron tablecloth and napkins	15min
Set table	20 min
Clean ups (do after each food item eg. stuffing, salad, pavlova and vegetables)	About 10 -15 min each time

PART C
Your Challenge

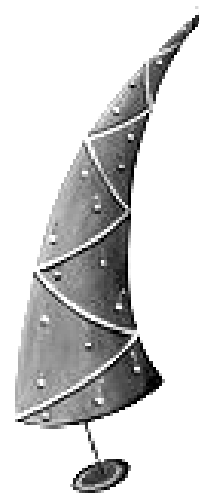
Dan, Sophie, and Eva are making Christmas dinner for their parents. From previous experience they know they need a well planned schedule that they can follow. The schedule needs to factor in all the jobs they need to do and the times they need to allow for each job. The schedule should also help them determine which job should be completed first in order to have all the food ready at the correct time.

In the kitchen, there are two kitchen benches to work on, a double sink, a microwave oven, and a stove with four top burners and an oven. The oven is large enough to fit the turkey and one other item at the same time.

The Thompson children are expecting their parents home at 6:30 pm. They want to serve nibbles first, the turkey as main course and then the pavlova for dessert.

Dan, Sophie, and Eva need your help!! They have so many tasks to complete to be ready to surprise their parents, and they need a reliable schedule. Can you help them do two things?

1. Make a preparation and cooking schedule. Chart what each person will do and when, including the use of the kitchen equipment.
2. Write an explanation of how you developed the schedule. They plan to have other surprise celebrations for their parents and want to use your explanation as a guide for making future schedules.



Appendix B

Schedule produced by the “Marvellous Mathematicians”

Serving Menu. 6.30 - cheese and crackers with vegetables and dip. Punch is also served at 6.30. 7 o'clock - warm bread. 7.15 - main course, which includes stuffed turkey, steamed vegetables, roast pumpkin and potato. 8 o'clock - fresh salad with dressing. 8.30 - pavlova with whipped cream and fresh strawberries.

Dan's schedule. Stuff turkey at 3.40. 3.45 - prepare turkey - put turkey in oven. 4.20 - wash and cut carrots, bean, broccoli, potatoes and pumpkin. 4.25 - steam carrots, bean and broccoli on stove. 4.50 - steam vegetables, off stove. 4.45 - mix gravy and put on stove. 5.25 - take roast vegetables out of the oven. 7.05 - take turkey out of the oven.

Sophie's schedule. At 2.45 - wash and cut ingredients for salad. 3 o'clock - make dressing and toss salad. At 3.30 cut and put bread on baking tray. 3.35 - warm bread in oven. 4 o'clock - separate eggs and whip egg whites for pavlova. 7 o'clock - bake pavlova in oven. 7.15 - whip cream wash and cut strawberries. 8 o'clock - take pavlova out of oven. 8.15 - prepare pavlova for serving.

Eva's schedule. At 4.20 - cut fruit, mix juice and soft drink. 4.30 - chill drinks for about 2 hours. 4.45 - wash and cut carrots into sticks. 5.10 - mix ingredients for dip and chill. 6.30 - take out drinks and dip. 6.30 - cut cheese and put out crackers.

All: Extra activities. Eva - punch clean up, cheese clean, carrot, broccoli, clean up, set table, dip clean, cleaning. Sophie - salad clean up, bread clean up, pavlova clean up, decorate house. Dan - clean up turkey, clean up vegetables, clean up gravy, clean house and iron tablecloth.

