THE IMPACT OF ZOLTAN DIENES ON MATHEMATICS TEACHING IN THE UNITED STATES

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I first encountered the writings of Zoltan Dienes as a young teacher of mathematics in a teacher education program. I had been prepared to be a teacher of mathematics in secondary school, but I was asked to teach some of the mathematics courses that would prepare teachers for elementary school. None of my preparation had introduced me to Dienes (I never saw anything by Bruner or Piaget either), and the prospective elementary teachers were asking questions about how children learn mathematics. This required attention to the structure of mathematics and to the psychology of learning mathematics. Dienes books and descriptions of lessons provided many of the answers I needed.

I first read *Building Up Mathematics* (Deines, 1971) when it was in its fourth edition (it was originally published in 1960). Here he described his theory of six stages of learning mathematics: (1) free play, (2) games, (3) search for communalities, (4) representation, (5) symbolization, and (6) formalization (p. 36). I also read *The Power of Mathematics* (Dienes, 1964), where he amplified his theories on structure and representation. His examples taught me several early lessons.

First, the role of play and games is crucial in formulating the first understanding of a new concept. Students can be introduced to very complicated ideas and can develop quite sophisticated approaches to problems if things are presented at the right level. Second, abstraction and generalization are important skills that must be practiced. This means finding several “embodiments” of mathematical concepts that children can explore. And third, a consistent Dienes claim, symbolism usually occurs too soon. When children are required to use symbols before they understand what the symbols represent, the learning involves mostly memory and is not very long lasting.

Several of Dienes’ inventions became standard equipment in the mathematics laboratory. His Multibase Arithmetic Blocks gave a concrete representation for number bases (see Figure 1 for a picture of the base 4 set). The principles of the base ten numeration system were so taken for granted that most students did not grasp the value of a base system. Dienes’ Blocks allowed students to explore the numeration system and how the operations on numbers are addressed by the system. Algorithms for addition, subtraction, multiplication and division can be illustrated and explained in detail, whether using the standard versions or alternative versions. Students were free to invent their own algorithms, and what’s more, they understood what they were doing.

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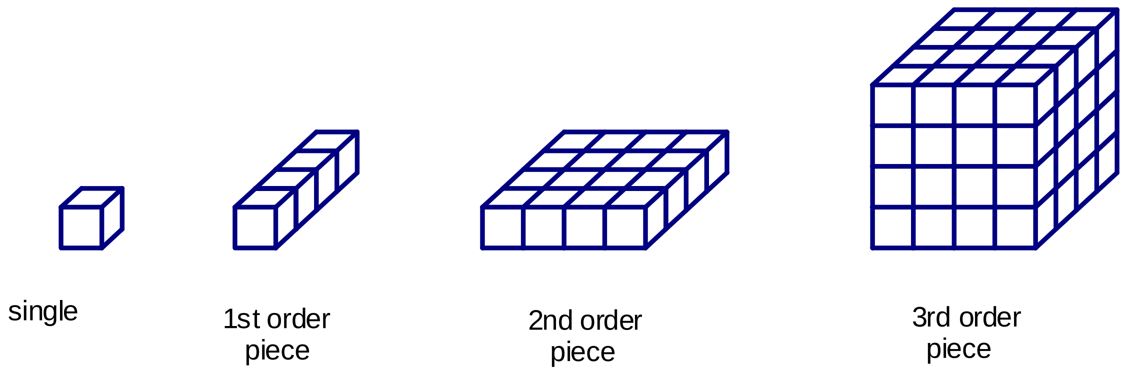

Figure 1: Base Four Arithmetic Blocks

A second set of materials from Dienes' fertile mind was the box of Logic Blocks (see *Learning Logic and Logical Games*, Dienes, 1974). The Logic Blocks were a completely balanced set of wooden pieces that varied in shape, color, size, and thickness. Every possible combination of the four attributes are available in the box. Examples of sets and their properties are easily shown; such as red pieces, non-red pieces, circle pieces, pieces that are red or circles, and so on. But just as important was the ability to define relationships between pieces; such as pieces that differ in one attribute, that differ in two attributes, that are the same shape, etc. Two pieces can be compared by "How are they alike?" or "How are they different?" Also, students can explore logical consequences in well-defined ways: "If a piece from this set is not a square, then it must be blue." Students who experienced activities using Dienes' materials often commented that the concrete examples made difficult concepts far more understandable. They supported Dienes' contention that trying to deal with mathematical symbolism (set notation) before the concepts were clear made learning difficult.

Not all of Dienes' examples are physical embodiments. One particular diagram (see Figure 2) is useful for several investigations of the additive structure or the multiplicative structure of number. For example, the horizontal (solid) arrow can represent "times 2" and the vertical (dashed) arrow can represent "times 3." With the leading circle set to 1, filling in the chart reveals many patterns of numbers, factors, and multiples. One can even go "backwards" along the arrows to see the relationships between fractions and inverses.

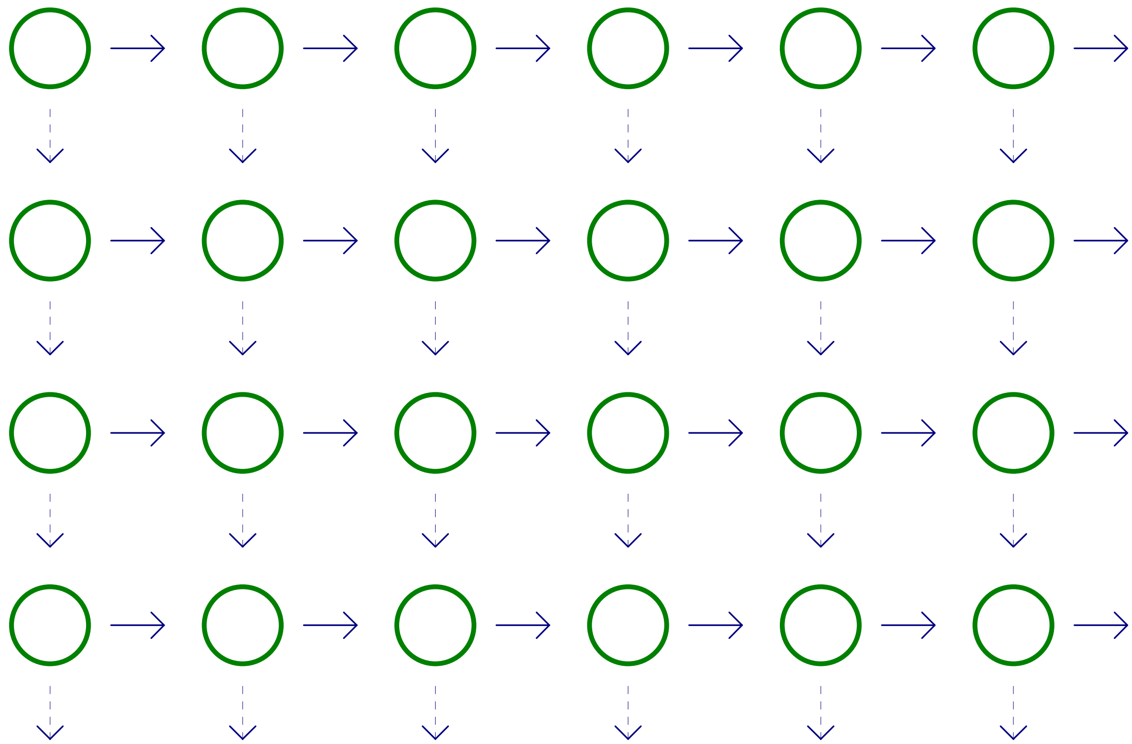

Figure 2: Diagram for Generating Elements (Dimension 2)

Dienes also gives many geometric illustrations. In a paper presented at a Symposium on Mathematics Laboratories (Dienes, 1975), he gives several concrete examples of finite geometries. As before, he gives multiple embodiments of the same structure and multiple representations that can be used to study geometric properties, such as duality and extensions. In his summary, he recapitulates his theory of the stages in this process:

> "In the abstraction process that leads to the eventual formation of a formalized concept, there are many stages. The first is always a somewhat groping stage, a kind of "trial and error" activity; this is usually described as <u>play</u>. The restrictions in the play lead to rule-bound play or <u>games</u>. This has been well represented in the present paper. The next stage is the identification of many different games possessing the same structure. This is the stage of the search for <u>isomorphisms</u>. When the irrelevant features of the many games have been discarded, we are ready for a <u>representation</u>. Such are the many "link" diagrams suggested. It is only when this stage has been reached that it is fruitful to use a fully symbolic language, the development of which will be a later stage in the abstraction process." (pp 83-84)

Have Dienes' ideas had an impact on school mathematics in the United States? At least indirectly, the influence has been major. Mathematics laboratories have been a part of many mathematics classrooms since the mid-70s, and hundreds of the activities were designed or inspired by Dienes' work. But in direct terms, one cannot be this positive. Most teachers would not recognize the contributions Dienes has made. Base 10 blocks may be common, but other bases are not available. So many versions of the Logic Blocks have been produced that Dienes rarely gets credit for these activities. Geometry lessons today deal more with physical descriptions and ignore logical arguments with abstract systems.

Part of the problem is due to timing and the cyclical nature of educational trends. Through the decade of the 1960s, the “new math” movement had gained, then lost, acceptance as a mathematics teaching method. This new math had emphasized abstract mathematics and formal justification. The method was generally suitable for good students, but it was found difficult for general audiences. Dienes theories were becoming known just as the new math was losing popularity. The U.S. mathematics curriculum was heading into a “back to basics” movement. Educators were concerned about school students’ abilities to perform calculations quickly and accurately. Dienes was describing processes like abstraction and generalization and justification (all are clearly considered basic to a mathematicians’ work, but general educators did not understand his mathematical examples). While teacher education programs may have tried to explain Dienes’ learning principles, only the simplest examples were meaningful to students (prospective teachers) with limited mathematical knowledge.

But Dienes’ message has been consistent. He has managed to create teaching activities that conform to his learning principles. His search for embodiments of mathematical ideas has produced clever examples in a wide variety of contexts. Music, motion, physics, dance, language, and even abstract games—they all conform to the similarities of the abstraction process. Dienes has shown that most children are capable of learning these sophisticated processes. The progression from concrete, through other representations, to symbols and formal structures applies to all areas of knowledge. Dienes’ great contribution has been that he has provided evidence of his principles at all levels and his activities use mathematical concepts. In many respects, it is best to demonstrate these processes in mathematics, so Dienes’ examples will continue to inspire mathematics teachers and cognitive scientists for years to come.

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