In this lab we will use the web resource located at http://www.aw-bc.com/ide. Once you are there click into the site and look for Part II Second Order Differential Equations, Lab 10, Free Vibrations. We will use both the damped vibrations tool, and the critical camping tool.

The ODE for free vibrations can be rescaled to look like this:

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + x = 0.$$

The roots of the characteristic equation are then

$$\lambda = -b \pm \sqrt{b^2 - 1}.$$

In the following exercises we will be varying the parameter b.

Open the damped vibration tool. Its purpose is to help you gain some intuition about the meaning of undamped, under-damped, critically damped and over-damped vibrations. The slider allows you to set the parameter b. Notice the graph of the eigenvalues, by this they mean roots of the characteristic equation for the ODE governing the oscillator. They can be real or complex, so we plot them in the complex plane.

1) Move the slider all the way to the left so that $b = 0$. This is the familiar case of a simple harmonic oscillator without damping. The eigenvalues are $\lambda = \pm i$ as shown in the display. These eigenvalues are also plotted as dots in the top panel, which shows their location in the complex λ plane. Describe how the eigenvalues change as you drag the slider to the right, increasing b.

2) The tool also shows the solution $x(t)$, starting from an initial condition $x(0) = 1, \dot{x}(0) = 0$. The corresponding motion of the mass is shown in the animation alongside the graph of $x(t)$. Notice that the time series and the animation have the same vertical scale: the current position of the mass is shown as a moving yellow dot on the graph of $x(t)$. The gray curves show the "envelope"- the time series stays between these curves. Move the slider towards $b = 1$. How do the vibrations change as b approaches 1 from below? What is the qualitative difference between the solutions for $0 \leq b < 1$ and $b \geq 1$?

3) Open the critical damping tool. It allows you to compare the vibrations for different values of b. All the solutions start from the same initial condition $x(0) = 1, \dot{x}(0) = 0$. The reference curve shown on the graph is the solution for the critically damped case, which has a special property, as you are going to see.

Move the slider for b. Notice how the solutions for different b compare to the critically damped case. By experimenting, find the value of b such that the solution $x(t)$ gets small and stays small as rapidly as possible.

Give a mathematical explanation of the previous answer. Why does that value of b give the fastest decay? (Hint: think about the magnitude of the eigenvalue at that value of b.)