Math 311 Fall 2011
Practice Test 2 Problems

1. Solve the following initial value problems. Write in phase amplitude form where appropriate and identify the envelope of the solution.
   a) $y'' = -2y' + y; \quad y(0) = 0; \quad y'(0) = -1.$
   b) $2w'' - 3w' + w = 0; \quad w(0) = 2; \quad w'(0) = 1.$
   c) $w'' + 9w = 0; \quad w(\pi) = 3; \quad w'(\pi) = -9.$
   d) $4x'' + 8x' + 5x = 0; \quad x(\pi) = 0; \quad x'(\pi) = -4.$

2. Solve the following non-homogeneous ODEs using annihilators (if needed) and undetermined coefficients.
   a) $y'' - 4y = 2\cos x + 2e^{2x}$
   b) $y'' + 9y = 5\cos 2x$
   c) $y'' + y' - 2y = -10\sin x$
   d) $y'' + 2y' - 3y = 8e^{x} - 12e^{3x}$

3. A force of 3 N stretches a spring by 1 meter.
   a) Find the spring constant k.
   b) A mass of 4 kg is attached to the spring. At $t = 0$ the mass is pulled down a distance 1 m from equilibrium and released with a downward velocity of 0.5 m/s. Assuming that the damping is negligible, determine an expression for the position of the mass at time t. Find the circular frequency of the system and the amplitude, phase, and period of the motion.

4. Consider the mass spring system whose motion is governed by the ODE

$$\frac{d^2y}{dt^2} + 2\alpha\frac{dy}{dt} + y = 0.$$

   Determine all the values of the positive constant α for which the system is (i) underdamped, (ii) critically damped, and (iii) overdamped. In the case of overdamping solve the system completely. If the initial velocity of the system is zero, determine whether the mass passes through equilibrium.

6. Circle *True* if the statement is true in all cases and *False* if it is not.

a) *True* *False* The phase amplitude form of the solution $x(t) = -4\cos t + 3\sin t$ is $5\cos(t - 2.5)$.

b) *True* *False* The two functions: $\cos t$ and $\cos(t - 2\pi)$ are linearly independent solutions to $x'' + x = 0$.

c) *True* *False* The phase lag for the steady state solution to $x'' + x' + x = \sin 2t$ is 0.317 radians (approximately).

d) *True* *False* The Wronskian of two solutions to a second order, linear, homogeneous ODE $\{y_1(t),\ y_2(t)\}$ is zero at every point in the interval of existence of the solutions if y_1 is not a constant multiple of y_2.