Math 311 Fall 2011
Practice Problems for the Final Exam

1. Find the general solution of the following first order ODE.

$$\frac{dy}{dx} = \frac{y}{x-2}$$

2. Solve the following initial value problem.

$$y' + 2x^{-1}y = 4x; \quad y(1) = 2.$$

3. Consider the mass spring system whose motion is governed by the ODE

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 17\sin 2t; \quad y(0) = -2, \frac{dy}{dt}(0) = 0.$$

   Determine the solution to the IVP and identify the transient and the steady state parts.

4. Consider the following nonhomogeneous second order linear ODE.

$$y'' + 2y' - 3y = 8e^x - 12e^{3x}$$

   a) First write the associated homogeneous problem in operator notation and factor it.

   b) Next find the solution to the associated homogeneous problem, y_c.

   c) Now find the annihilator of the right hand side $8e^x - 12e^{3x}$.

   d) Use the annihilator to find the form of the particular solution, y_p. **DO NOT SOLVE FOR THE COEFFICIENTS**.

5. Consider the following set of two coupled first order linear constant coefficient ODEs.

$$x_1' = 9x_1 + 5x_2$$

$$x_2' = -6x_1 - 2x_2$$

   a) Write it as a matrix vector equation, $\mathbf{x}' = A\mathbf{x}$.

   b) Compute the eigenvalues and eigenvectors of A.

   c) From this write the vector valued general solution to the system.

   d) Finally, find the solution with the initial condition $x_1(0) = 1; x_2(0) = 0$.

6. For each of the following two dimensional systems, determine the type and stability of the fixed point at the origin.

   a) $x' = x$
      $y' = -2y$

   b) $x' = 2x$
      $y' = 3y$

   c) $x' = 3x - 2y$
      $y' = 4x - y$

   d) $x' = x - 2y + 3xy$
      $y' = 2x - 3y - x^2 - y^2$

7. Circle *True* if the statement is true in all cases and *False* if it is not. Show work as needed for partial credit.

   a) *True* *False* $\lambda = 0$ is an eigenvalue for the boundary value problem $y'' + \lambda y = 0;\ \ y(0) = 0;\ y(1) = 0.$

   b) *True* *False* The Existence and Uniqueness theorem does not guarantee a unique solution to this initial value problem: $y' = \tan y;\ \ y(0) = 1.$

   c) *True* *False* The critical points for the autonomous system $x' = x(x-1) + y, y' = 3y$ are $(0, 0)$ and $(1, 0)$.

   d) *True* *False* The mass-spring system governed by $x'' + 2x' + 9x = \cos(3t)$ exhibits the phenomenon of resonance.

   e) *True* *False* The solution, $x(t)$, to $x'' - x = 0$ oscillates at constant amplitude for all t.