Due: Tuesday, December 15, 10PM.

1. Suppose that $X \sim \text{Bin}(5, .5)$.

a. Give the probability distribution $F(x)$ of X.

b. Compute the expectation and standard deviation σ of X. You may use the fact that for a binomial random variable, $\mu = EX = np$ and $\sigma^2 = np(1-p)$.

c. Find the probability function and probability distribution of $Y = |X - \mu|$.

d. Find EY.

e. Find $\text{Var}(Y)$.

2. In the mid 1980's it was suggested that the entire U.S. population be tested for the presence of HIV. Would this be successful? At the time, the ELISA test had a sensitivity of 95% and a specificity of 99%. The sensitivity of a test is the proportion of people with the disease who will test positive, and the specificity of a test is the proportion of people without the disease who will test negative. The prevalence of AIDS was estimated to be about 1 in 1000 at the time.

a. A false positive is a positive test for someone that does not have HIV. What is the probability that the test of a random selected person will be a false positive?

b. A false negative is a negative test for someone that does have HIV. What is the probability that the test of a randomly selected person will be a false negative?

c. Calculate the probability that a person has HIV given a positive ELISA test result.

d. Calculate the probability that a randomly tested person does not have HIV given a negative ELISA test result.

e. In light of your answers to parts a. and b., do you think that testing the entire population would be acceptable?

3. Suppose that 30% of the people in a certain population have blood type A.

a. Find the probability that the first person with type A will be the third tested in sequence of randomly selected people. (Think of a line of army recruits being tested for blood type.)

b. Find the probability that exactly one person in among the first three tested will have type A.

c. Identify the distribution of X, the number of people among the first three with type A. (That is, it is not necessary to compute the probabilities if you can identify the distribution).

d. What is the expected number among the four with type A?

4. An urn contains 7 green and 3 red balls. Three balls are drawn from the urn at random and without replacement.

a. What is the probability that all 3 balls are red?

b. Let Y denote the number of red balls among the three that are drawn. Find the probability distribution of Y.

c. Find the distribution function of Y.

d. The distributions of X (problem 3) and Y (this problem) are different because a key characteristic of the random experiments is different. What is this difference?

5. A large herd of cattle is to be checked for bovine spongiform encephalopathy (BSE), commonly known as mad-cow disease, by looking for two symptoms of BSE. It is thought that 20% of cattle with advanced BSE exhibit symptom A (nervous or aggressive behavior) alone, 30% of cattle with advanced BSE exhibit symptom B (difficulty standing) alone, 10% of cattle with advanced BSE exhibit both symptoms, and the remainder of infected cattle exhibit neither symptom. Suppose that the entire herd is infected with advanced BSE.

a. What is the probability that a randomly selected cow exhibits neither symptom?

b. What is the probability that a randomly selected cow exhibits at least one symptom?

c. What is the probability that a randomly selected cow exhibits both symptoms given that it exhibits symptom A?