1. Suppose that $X \sim \text{Poisson}(\lambda = 5)$. Hence

$$F(x) = \begin{cases} 0, & \text{for } x \le 0, \\ 1 - \exp(-\lambda x) & \text{for } 0 < x. \end{cases} \tag{1}$$

a. Find p_{25} and p_{75}, the 25th and 75th percentiles of the distribution of X.

b. Suppose that $Y = r(X)$ is defined by

$$Y = \begin{cases} 1, & \text{for } x \le p_{25}, \\ 2 & \text{for } p_{25} < x \le p_{75} \\ 3 & \text{for } x < p_{75} \end{cases}$$

c. Find the probability function of Y.

d. Graph the distribution function of Y.

e. Find a median of the distribution of Y.

2. Let $U \sim \text{unif}(0, 1)$.

a. Compute EU and $E(U^2)$

b. Compute $\sigma^2 = \text{var}(U)$.

c. Compute $P(|Y - \mu| \le 2\sigma)$.

d. According to Chebychev's inequality, $P(|Y - \mu| \le 2\sigma)$ is no less than what number?

3. Suppose that $X \sim \exp(\lambda)$ with $\lambda = 1/10$.

a. Compute EX.

b. What is the variance of X?

c. Compute $P(|Y - \mu| \le 3\sigma)$.

d. According to Chebychev's inequality, $P(|Y - \mu| \le 3\sigma)$ is no less than what number?

4. Assume that the random variable Y has a distribution given by formula (1).

a. Compute $P(1 \le Y \le 3)$.

b. Let the events A and B be defined by $A = \{Y \ge 1\}$ and $B = \{Y \le 3\}$. Write an expression for $A \bigcap B$ in terms of Y.

c. Compute $P(A \bigcap B)$.

d. Compute $P(Y \ge 1 | Y \le 3)$.