1a. Given a league with 9 teams, $A, B, C, \ldots, I$, how many games must be played in order that each team play every other team once? $C_{9,2} = 36$

1b. How many games does team A play in? 8

1c. How many games must be played in order that each team play every other team five times? $5C_{9,2} = 180$

2. Suppose a racing car event has 30 cars entered. Of these, 20 use motor oil A, 6 use motor oil B, and 4 use motor oil C.

- **a.** Keeping track only of the type of motor oil used, how many different orderings could be observed as the cars pass the finish line? $\frac{30!}{20!6!4!}$
- **b.** Assume that all the cars, drivers and motor oils are equally good, so that each ordering is equally likely to be observed. What is the probability that the winning car uses oil A? 20/30
- **c.** Again assuming that each ordering is equally likely, what is the probability that the first two finishers use oil A? $\frac{20}{30} \cdot \frac{19}{29} = 0.43678$
- **d.** What is the probability that all first three finishers use oil C? $\frac{4}{30} \cdot \frac{3}{29} \cdot \frac{2}{28} = 0.000985$

3. A car salesperson assumes that her sales of cars appear to be Poisson in distribution, and that the mean number of cars sold per week is about 2.

- **a.** Evaluate the probability that she makes no sales in a week. $e^{-2} = 0.13533$
- **b.** Evaluate the probability that she makes 2 sales in a week. $\frac{2^2 e^{-2}}{2!} = 0.27067$
- **c.** Evaluate the probability that she makes at least 2 sales in a week. $1 - (e^{-2} - 2e^{-2}) = 1 - (0.13533 + 0.2707)$.
- **d.** What is the standard deviation of the number of sales per week? $\sqrt{\text{var}(X)} = \sqrt{2}$, where X is a random variable representing the number sold in a future week.

4. Two missiles, A then B, are fired at a target. Missile A will hit the target with probability 0.2 and B will hit the target with probability 0.2 if A misses the target. If A hits the target, then the probability that B hits the target is .6. Let X denote the number of missiles that hit the target.

- **a.** Find the probability distribution of X.

x	0	1	2
$P(X = x)$	.64	.24	.12

- **b.** Find the expected number of hits. $EX = .48$

4. Assume that a test for cancer is correct 90% of the time (that is, it correctly detects the presence of cancer with probability .9 and correctly identifies the absence of cancer with probability .9). Assume also that 5% of the population that is tested for the cancer actually has the cancer. Of those individuals that take the test and are said to have cancer according to the test, what proportion actually have the cancer?

Let C denote the event that an individual has cancer so that $P(C) = .05$; let D denote the event that the test detects cancer so that $P(D|C) = P(D^c|C^c) = .9$. Then, the desired proportion is $P(C|D) = \frac{.05 \cdot .9}{.05 \cdot .9 + .95 \cdot .1} = .321$

5. (This problem is more challenging than you should expect on the exam; however, it is a good review problem.) Show that $P(A \bigcup B) = P(A) + P(B)P(A^c|B)$. First,

$$\begin{aligned} P(A) + P(B)P(A^c|B) &= P(A) + P(A^c \bigcap B). \\ &= P[A \bigcup (A^c \bigcap B)] \end{aligned}$$

since $A \bigcap (A^c \bigcap B) = \emptyset$. Since $A \bigcup (A^c \bigcap B) = A \bigcup B$, $P(A \bigcup B) = P(A) + P(B)P(A^c|B)$.