1. Suppose that a random variable X can have each of seven values $-3, -2, -1, 0, 1, 2, 3$ with equal probability. Determine the probability function and distribution function of the random variable $Y = X^2 - X$.

y	0	2	6	12
$P(Y = y)$	2/7	2/7	2/7	1/7
$P(Y \leq y)$	2/7	4/7	6/7	1

2. Suppose that the density function of the random variable X is

$$f(x) = \begin{cases} \frac{1}{2}x & \text{for } 0 < x < 2, \\ 0 & \text{otherwise.} \end{cases}$$

Also, suppose that $Y = \log(X)$. Determine the density and distribution functions of Y.

$$F_Y(Y) = \begin{cases} \frac{\exp(2y)}{2} & \text{for } -\infty < y \leq \log(2), \\ 0 & \text{otherwise,} \end{cases}$$

and

$$f_Y(Y) = \begin{cases} \frac{\exp(2y)}{4} & \text{for } -\infty < y \leq \log(2), \\ 0 & \text{otherwise.} \end{cases}$$

3. Suppose that the density function of X is as given in exercise 2. Suppose the random variable $r(X) = Y$ is defined by

$$r(x) = \begin{cases} 1 & \text{if } 0 < x < .5, \\ 2 & \text{if } .5 \leq x < 1, \\ 3 & \text{if } 1 \leq x < 1.5, \\ 4 & \text{if } 1.5 \leq x < 2, \\ 0 & \text{otherwise.} \end{cases}$$

a Find the probability function $P(Y = y)$.

y	1	2	3	4
$P(Y = y)$	1/16	3/16	5/16	7/16

b Find the distribution function $F_Y(y)$.

y	1	2	3	4
$P(Y \leq y)$	1/16	4/16	9/16	1

c Find EY and EX. $EY = 50/16 = 3.125$ and $EX = \int_0^2 \frac{x^2}{2} \mathrm{d}x = 8/6$.

4. The waiting time until the delivery of a computer component is uniformly distributed over the interval from 1 to 5 days. The cost of the delay in dollars is given by $Y = 3U^2 + 1$, where U is the waiting time in days. The distribution function of U is

$$F_U(u) = \begin{cases} 0 & \text{for } u \leq 1, \\ \frac{u-1}{4} & \text{for } 1 < u < 4, \\ 0 & \text{for } 4 \leq u. \end{cases}$$

a Find f_U.

$$f_U(u) = \begin{cases} \frac{1}{4} & \text{for } 1 < u < 4, \\ 0 & \text{otherwise.} \end{cases}$$

b Find the distribution function for Y.

$$F_Y(Y) = \begin{cases} 0 & \text{for } y \leq 4 \\ \frac{\sqrt{\frac{y-1}{3}}}{4} & \text{for } 4 < y \leq 76, \\ 1 & 76 < y. \end{cases}$$

c Show that there is a .5 probability that the cost will exceed 28. Hint: compute $P(Y \leq 28) = \frac{1}{2}$.
Evaluate $F_Y(28)$:

$$P(Y \leq 28) = F_Y(28) = \frac{\sqrt{\frac{28-1}{3}}}{4} = \frac{1}{2}.$$