25. $X = \#$ of 3's $\sim \text{Bin}(8, 1/6)$. Then,

$$P(X=2) = C_{8,2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^6 = \frac{8!}{2!6!}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^6 = .26047.$$

27. Let W denote the outcome of team B winning a game. Then, B will win the series if B wins the first 4 games, or 3 of the first 4 games and the fifth game, of 3 of the first 5 games and the sixth game, or 3 of the first 6 games and the seventh game. The probability of each of these outcomes is computed using the binomial distribution, and the sum of the probabilities is the probability of winning the series. Thus,

$$\begin{aligned} P(\text{win series}) &= C_{4,4}.6^4.4^0 + C_{4,3}.6^3.4^1 \times .6 + C_{5,3}.6^3.4^2 \times .6 + C_{6,3}.6^3.4^3 \times .6 \\ &= .6^4 + .6^4 \cdot 4 \cdot .4 + .6^4 \cdot 10 \cdot .4^2 + .6^4 \cdot 20 \cdot .4^3 \\ &= 0.71021. \end{aligned}$$

28a. $X \sim \text{Bin}(20, .108)$, so $P(X=2) = C_{20,2}.108^2.892^{18} = 0.2832.$

28b. $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = 0.6312.$

35. To use the Poisson approximation, we need to compute $\lambda = np$; for parts a,b, and c, the values of λ are all 1. The R functions to compute probabilities for a vector of values such as $x = (0, 1, 2, \ldots, n)$ are pbinom(x,size=n,prob=p) and ppois(x,lambda=np). The plotted values are shown below:

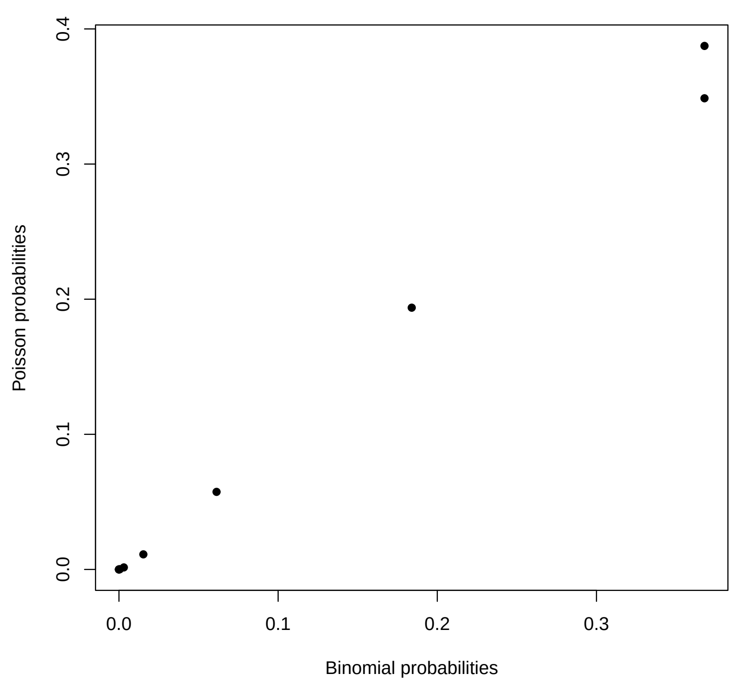

Figure 1: Probabilities for a).

37. Let $y \sim \text{Pois}(\lambda = 2/5)$ since the described random variable is binomial with $n = 20$ and $p = 1/50$. Then $P(Y \geq 1) = 1 - P(Y=0) = e^{-.4} = 0.32968.$

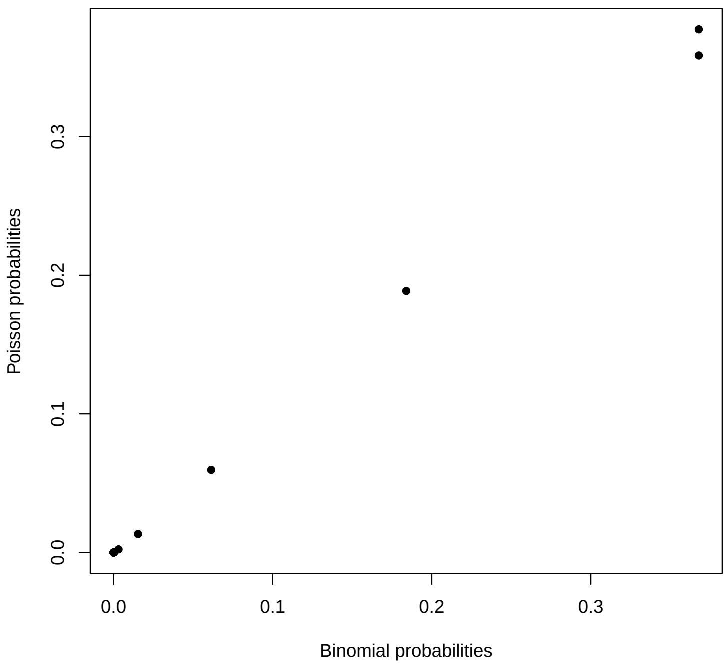

Figure 2: Probabilities for b).

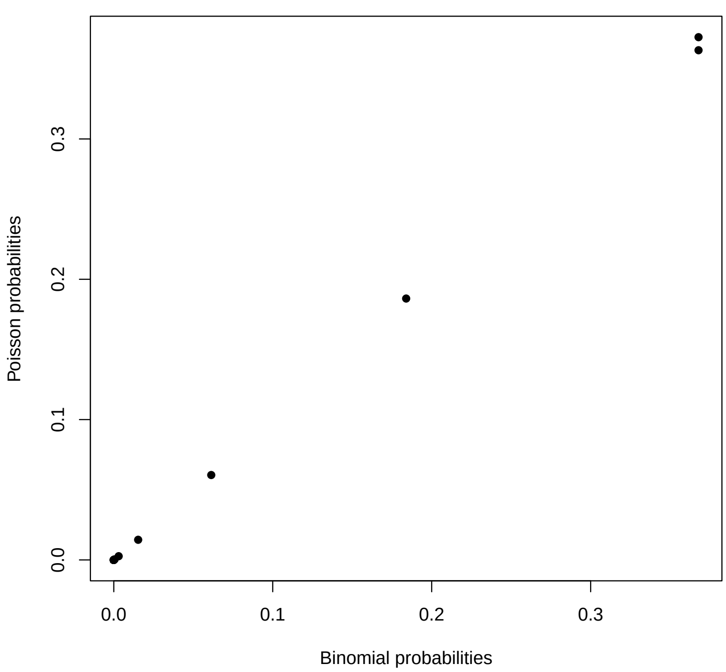

Figure 3: Probabilities for b).

R Problem The figure shows the distribution of sample means computed from 10000 samples of size $n = 25$ randomly drawn from the set of all ages. The observed sample mean of those individuals that died was 35.48, and this value is as larger or larger than only 128 of the 10000 simulated means. It is highly improbable to obtain such a large sample mean if there were no real difference in ages between survivors and those that died. Thus, I conclude that the difference is real, and that older individuals were less likely to survive than younger individuals.

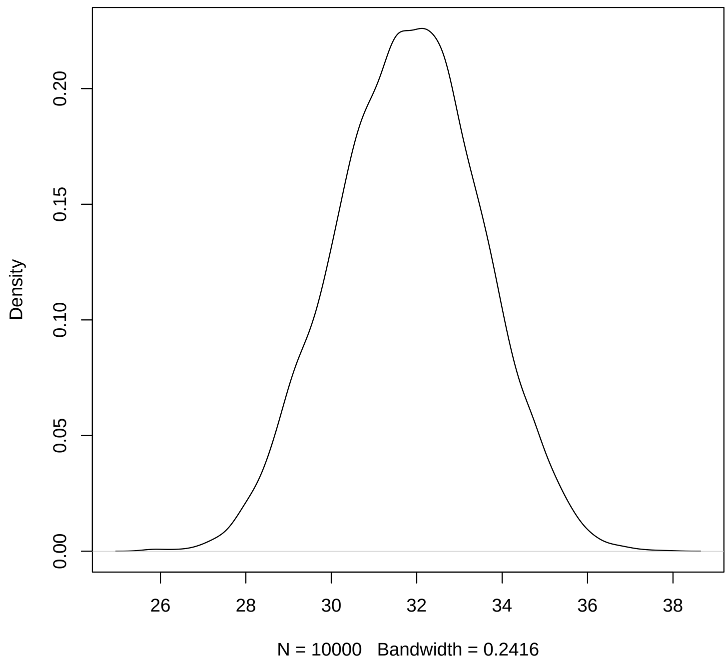

Figure 4: Distribution of 10000 simulated means.