20. A and B are independent. To show, we need to compute $P(A)$, $P(B)$ and $P(A \bigcap B)$. First $P(A) = 4/52$. The event $B =$ "spade on the second draw" will happen if we observe a spade on both the first and second draws, or if we observe some other suit besides spade on the first and a spade on the second draw. Thus,

$$P(B) = \frac{13 \cdot 12}{52 \cdot 51} + \frac{49 \cdot 13}{52 \cdot 51} = \frac{13}{52}.$$

The event $A \bigcap B$ will occur if we observe the ace of spades on the first draw and a spade on the second draw, or if we observe an ace that is not a spade on the first draw and a spade on the second. Thus,

$$P(A \bigcap B) = \frac{1 \cdot 12}{52 \cdot 51} + \frac{3 \cdot 13}{52 \cdot 51} = \frac{1}{52}.$$

Now we compare $P(A)P(B)$ to $P(A \bigcap B)$: $P(A)P(B) = \frac{4}{52}\frac{13}{52} = \frac{1}{52} = P(A \bigcap B)$. Hence, A and B are independent events.

23. Note that $P(A) = 18/36 = 1/2$, $P(B) = (2+5+4+1)/36 = 12/36$, and $P(A \bigcap B) = 6/36$. Since $P(A \bigcap B) = 6/36 = 1/2 \times 12/36 = P(A)P(B)$, independence holds.

29. $P(A \bigcap B^c \bigcap C^c) + P(A^c \bigcap B \bigcap C^c) + P(A^c \bigcap B^c \bigcap C) = 1/4 \times 2/3 \times 1/2 + 3/4 \times 1/3 \times 1/2 + 3/4 \times 2/3 \times 1/2 = 11/24$.

33. The rolls are independent, so the probability of at least one 6 in 4 rolls is the 1 minus the probability of no sixes in 4 rolls. The probability is $1 - (5/6)^4 = 0.5177$. Similarly, the probability of at least one double 6 in 24 rolls is $1 - (35/36)^{24} = 0.4914$.

35. Refer to 5, and recall the table of absolute differences:

0	1	2	3	4	5
1	0	1	2	3	4
2	1	0	1	2	3
3	2	1	0	1	2
4	3	2	1	0	1
5	4	3	2	1	0

As each pair is equally likely to occur, the probability of each absolute difference is the relative frequency of the absolute difference in the table; hence, the distribution is

x	0	1	2	3	4	5
$P(X = x)$	6/36	10/36	8/36	6/36	4/36	2/36

38. Suppose that the number of children is n, and X is the number of female children. The event "at least one child of each sex" is equivalent to the event $A_n = \{0 < X < n\}$. We need to try different values of n and compute $P(A_n)$, or more simply, $1 - P(A_n^c)$ since $A_n^c = \{X = 0\} \bigcup \{X = n\}$. Now,

$$P(A_n^c) = 2 \times \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{n-1},$$

assuming that males and females are equally likely and that births are independent events. Using R to compute powers of 1/2, I find that

$n-1$	2	3	4	5
$P(A_{n-1}^c)$	0.25	0.125	0.0625	.03125

from which it is apparent that $n - 1 \geq 5$ is required, and that the couple should have at least 6 children.