Chapter 10 Problems

10.18. **a)** $\widehat{\pi} = 35/800 = 0.04375$. Note $n\widehat{\pi} = 35 > 5$ and $n(1-\widehat{\pi}) = 800 - 35 > 5$. Hence, the normal approximation is accurate and appropriate for these data.
b) $H_0 : \pi = 0.096$ $H_a : \pi < 0.096$. Let $\pi_0 = 0.096$. The test statistic is

$$Z = \frac{\widehat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}.$$

Plugging in π, $\widehat{\pi}$ and n gives

$$Z = \frac{0.0438 - 0.096}{\sqrt{\frac{0.0438(1-0.0438)}{800}}} = \frac{-0.0522}{0.0104} = -5.01.$$

If $H_0 : \pi = 0.096$ is true, then Z is approximately $N(0,1)$ in distribution. Given $\alpha = 0.05$, a rejection region for a test of H_0 is $\{z\ z < -1.96\}$. Clearly, there is abundant evidence against H_o and favoring H_a.

10.34. **a**) We want to test $H_0 : \pi_1 - \pi_2 = 0$ versus $H_a : \pi_1 - \pi_2 > 0$ where π_1 and π_2 are for Minoxidol and placebo groups respectively. Here, $\widehat{\pi}_1 = 32/310 = 0.1032$ and $\widehat{\pi}_2 = 20/310$. Also $\widehat{\pi} = (y_1 + y_2)/(n_1 + n_2) = (20 + 32)/(310 + 310) = 0.0839$. I prefer to use the test statistic

$$Z = \frac{\widehat{\pi}_1 - \widehat{\pi}_1 - (\pi_1 - \pi_2)}{\sqrt{\widehat{\pi}(1-\widehat{\pi})(\frac{1}{n_1} + \frac{1}{n_2})}}.$$

(You may also use the test statistic given on p. 484 of the text). The value of my test statistic is

$$Z = \frac{0.1032 - 0.0645}{\sqrt{0.0839(1-0.0839)(\frac{1}{310} + \frac{1}{310})}} = \frac{0.0387}{0.0222} = 1.74.$$

Large values of $\widehat{\pi}_1 - \widehat{\pi}_1$ support H_a, as do large values of Z; hence p-value $= P(Z \geq 1.74) = 0.0409$. There is evidence of a difference in pi's between the two populations.
b) The extent of new hair growth is important to estimate; also, did the study reveal any possible side effects of the treatment?

10.35. **a**) We want to test $H_0 : \pi_1 - \pi_2 = 0$ versus $H_a : \pi_1 - \pi_2 > 0$ where π_1 and π_2 are for cocaine and heroin groups respectively. Here, $\widehat{\pi}_1 = 0.9$ and $\widehat{\pi}_2 = 0.36$. Also $\widehat{\pi} = (90 + 36)/200 = 126/200 = 0.63$. I prefer to use the test statistic

$$Z = \frac{\widehat{\pi}_1 - \widehat{\pi}_1 - (\pi_1 - \pi_2)}{\sqrt{\widehat{\pi}(1-\widehat{\pi})(\frac{1}{n_1} + \frac{1}{n_2})}}.$$

(You may also use the test statistic given on p. 484 of the text). The value of my test statistic is

$$Z = \frac{0.9 - 0.36}{\sqrt{0.63(1-0.63)(\frac{1}{100} + \frac{1}{100})}} = \frac{.53}{0.0683} = 7.90.$$

Large values of $\widehat{\pi}_1 - \widehat{\pi}_1$ support H_a, as do large values of Z; hence p-value $= P(Z \geq 7.90) < 0.0001$. There is very strong evidence of a difference in pi's between the two populations.

b) Implications are unclear given differences between humans and rats. Moreover, the situation is unrealistic because few drug users have unlimited access to illegal drugs.

10.46. **a**) The hypotheses of interest are $H_0 : \pi_1 = 0.4 = \pi_2, \pi_3 = 0.1 = \pi_4$ versus H_a : not H_0. The test statistic is the χ^2 goodness of fit test, and the relevant information for computing the test are tabled:

Season	n_i	E_i	$\frac{(n_i - E_i)^2}{E_i}$
Winter	374	400	1.70
Spring	292	400	29.7
Summer	169	100	47.6
Fall	165	100	42.2
Total	1000	1000	120.7

Note that the differences between the expected counts E_i and the observed counts n_i are very large for all seasons but winter, and so the doctor's model is very poor (based on the observed data). Consequently, the chi-square statistic $\chi^2 = 120.7$ is extremely large if H_0 were true. Under H_0, χ^2 has an approximate chi-square distribution with 4 degrees of freedom. The rejection region are all values of χ^2 greater than 9.488 (Table 7 in the text). Hence, I reject H_0 in favor of H_a.