Final Review Problems

p. 284. 6.9 a) The normal distribution assumption appears to be justified by the P-P plots below.

Control group

![Normal P-P Plot of CONSUME](image)

Expected Cum Prob

Observed Cum Prob

Experimental group

![Normal P-P Plot of CONSUME](image)

Expected Cum Prob

Observed Cum Prob

p. 284. 6.9 b) $H_0 : \mu_C = \mu_E$ versus $H_a : \mu_C < \mu_E$, where $\mu_C$ and $\mu_E$ are the population mean amount of lead acetate consumed by the population of control and experimental animals, respectively. Because $s_1 = 1.18$ and $s_2 = 1.47$ are approximately equal, I will use the pooled two-sample $t$-test. The value of the test statistic is $T = -5.85$ and $P(T \leq -5.85) < 0.002$ with $df = 18$. Hence there is sufficient evidence to reject $H_0$ in favor of $H_a$, given $\alpha = 0.05$.

p. 285. 6.12) I do not think the data are normal in distribution; instead, they are likely to be skewed. For the magnesium data, $\bar{Y} = 1.0$ and $s = 2.21$. According the the empirical rule, if these data are from a normal distribution, then about 17% of the distribution will be less than $\bar{Y} - s = 1.0 - 2.21 = -0.79$. But none of the observations on concentration are less than 0, so these data must be skewed.

p. 285. 6.13) $t$-procedures are not appropriate, given that the data are apparently not normal in distribution, and the sample sizes are relatively small. If the data did appear normal, then the $t$-procedures would be O.K., after adjusting for scale differences.
p. 296. 6.20a) The normal distribution assumption appears not to be justified by the P-P plots below.

<table>
<thead>
<tr>
<th>Treatment Group</th>
<th>Control Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal P-P Plot of SB2M</td>
<td>Normal P-P Plot of SB2M</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Observed Cum Prob</th>
<th>Expected Cum Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>0.25</td>
<td>0.25</td>
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<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The boxplot provides ambiguous evidence that the populations are skewed.

p. 296. 6.20b) Wilcoxon Rank Sum test is preferred to the $t$-procedures.

p. 296. 6.20c) Wilcoxon Rank Sum test yields a value of $T = 93$, and a $p$-value of 0.393 for a test of $H_0$: the two populations are identical, versus the alternative $H_a$: the populations are shifted relative to each other.
p. 296. 6.20d) The implications are that before treatment, the groups appear to be the same with respect to the distribution of SB2M. This is good, because if differences had been found, then it would be more difficult to isolate the effect of the drug in the analysis of the study results. It should be noted that this is an improper test in the sense that the experimenters' prefer not to reject \( H_0 \). If they were dishonest, then this outcome could be made more likely by testing a smaller subsample of all patients. This would reduce the power the test.

p. 305 6.27 a) I would limit the population by using only college students, or some other group that is relatively homogeneous with respect to age and health. Some additional screening could be carried out using an application form and asking if individuals were smokers or had chronic diseases. Body fat is more difficult; it can't be measured accurately without a substantial effort. Having collected a sample of 30 individuals, these would be randomly assigned to one of two groups (equal sizes).

The observed data would be pre- minus post-treatment change in cholesterol for two independent groups corresponding to drug type.

6.27 b) After collecting a random sample of 30 individuals, then I would pair them so that the members of a pair were as similar as possible with respect to age, health, etc. One member of each pair would be randomly assigned to the new drug, the other member would receive the other drug.

The observed data would be pre- minus post-treatment change in cholesterol for two independent groups corresponding to drug type. I would use a paired-data procedure to compare the reductions between group.

6.27 c) To determine which method is most effective, I would use the paired design and carry out the test of significance. Then, I would eliminate the pairing, treat the data as two independent samples, and carry out a test of significance using a independent-samples procedure. A comparison of observed significance level would give some information on the efficiency of designs.

p. 306, 6.29a) These data are paired. A test must be based on the differences, so I will use the 10 sample differences to assess whether the population of differences can be considered to be normal. The P-P plot below suggests that the true distribution of differences is not grossly different from the normal distribution assumption. So, it is weakly justifiable.
I will test $H_0 : \mu_D = 0$ versus $H_a : \mu_D > 0$, where $\mu_D$ is the population mean of the before – after variable. The test statistic is the paired $t$ with df $= n - 1 = 10 - 1 = 9$; the value of the test statistic is $T = 0.861$, df $= 9$, and $p$-value $= .412$. There is no evidence of supporting $H_a$.

6.29b) A 95% CI for $\mu_D$, based on the pooled two-sample $t$ statistic is, $-4.9$ to $9.4$.

6.29c) Conditions require for normal distribution-based procedures seem to be satisfied

p. 312. 6.33a) I will test the hypotheses $H_0$: the distribution of differences is symmetrically distributed about 0, versus $H_a$: the distribution of differences is distributed about some value besides 0.

The value of the Wilcoxon Signed Rank test statistic is $T = 3$ and the approximate $p$-value is $0.24$. There is no evidence of supporting $H_a$.

6.33 b) It does not matter whether the paired $t$ or Wilcoxon signed rank test statistic is used—they lead to the same conclusion.

6.73 a) $H_0 : \mu_1 - \mu_2 = 0$ versus $H_0 : \mu_1 - \mu_2 > 0$, where $\mu_1$ and $\mu_2$ are the population mean potencies when the bottle is drawn from current production, and when the bottle is shelved for 1 year, respectively.

6.73 b) The values are $t = 4.23$ and $t' = 4.23$. They are equal because the sample sizes are equal, which means that the pooled variance estimate of $\sigma_{\bar{y}_1 - \bar{y}_2}$, and the separate variance estimate of $\sigma_{\bar{y}_1 - \bar{y}_2}$ are equal.

6.73 c) The $p$-values are $0.0003$ and $0.00025$ for the $t'$ and $t$ statistics, respectively. These differ because the degrees of freedom are different.

6.73 d) Conclusions are the same regardless of the choice of the $t$ statistic.
6.73 e) $t$ is more appropriate, though the question is misleading. $t$ is better in general in these situations of near-equal population variances because it has greater power (simply because the degrees of freedom are greater). It would be better to ask "which is preferable?"