

# Part II

## Reflections on Theories of Learning

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# Preface to Chapter 2 Ernest's Reflections on Theories of Learning

## Lakatos–Hersh–Ernest: Triangulating Philosophy–Mathematics–Mathematics Education

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Philosophy has always maintained an intricate relationship with mathematics. It was also implicitly accepted that the philosophical positions of a bearer influence his/her view on mathematics and its teaching (Törner and Sriraman 2007), which leads us into the domain of beliefs theory. However the centrality of philosophy and its intricate relationship to theory development in mathematics education only came about two decades ago when Paul Ernest and Hans-Georg Steiner (1987) each independently became aware of the importance of epistemological issues that impact the teaching and learning of mathematics. Sierpiska and Lerman (1996) state:

Epistemology as a branch of philosophy concerned with scientific knowledge poses fundamental questions such as: ‘What are the origins of scientific knowledge?’ (Empirical? Rational?); ‘What are the criteria of validity of scientific knowledge?’ (Able to predict actual events? Logical consistency?); ‘What is the character of the process of development of scientific knowledge?’ (Accumulation and continuity? Periods of normal science, scientific revolutions and discontinuity? Shifts and refinement in scientific programs?).

The question of what is mathematics, for teaching and learning considerations brings into relevance the need to develop a philosophy of mathematics compatible with mathematics education. In order to answer this question for mathematics education, several theorists have played a role directly or indirectly. In this preface to Chap. 2, we briefly summarize the role that Lakatos, Hersh and Ernest have played. Reuben Hersh began to popularize Lakatos’ book *Proofs and Refutations* to the mathematics community in a paper titled, “Introducing Imre Lakatos” (1978) and called for the community of mathematicians to take an interest in re-examining the philosophy of mathematics. Hersh (1979) defined the “philosophy of mathematics” as the working philosophy of the professional mathematician, the philosophical attitude to his work that is assumed by the researcher, teacher, or user of mathematics and especially the central issue—the analysis of truth and meaning in mathematical discourse. Much later, Hersh (1991), wrote

Compared to “backstage” mathematics, “front” mathematics is formal, precise, ordered and abstract. It is separated clearly into definitions, theorems, and remarks. To every question there is an answer or at least, a conspicuous label: “open question”. The goal is stated at the beginning of each chapter, and attained at the end. Compared to “front” mathematics, mathematics “in back is fragmentary, informal, intuitive, tentative. We try this or that, we say “maybe” or “it looks like”.

47 So, it seems that Hersh is not too concerned with dry ontological concerns about  
48 the nature of mathematics and mathematical objects, but is more concerned with the  
49 methodology of doing mathematics, which makes it a human activity. In 1978 Paul  
50 Ernest published a review of *Proofs and Refutations* in *Mathematical Reviews*, and  
51 subsequently wrote reviews of the works of Lakatos and Wittgenstein (see Ernest  
52 1979a, 1979b, 1980). This coupled with his doctoral dissertation became the basis  
53 on which Ernest formulated a Philosophy of Mathematics Education (Ernest 1991)  
54 and Social constructivism as a philosophy of mathematics (Ernest 1998).

55 Pimm et al. (2008) summarize Ernest's "extension" of Lakatos' philosophical  
56 position as follows:

57 Ernest (1991) claimed that the fallibilist philosophy and social construction of mathemat-  
58 ics presented by Lakatos not only had educational implications, but that Lakatos was even  
59 aware of these implications (p. 208). Ernest argued that school mathematics should take  
60 on the socially constructed nature presented by Lakatos, and also that teacher and students  
61 should engage in ways identical to those in his dialogue, specifically posing and solving  
62 problems, articulating and confronting assumptions, and participating in genuine discus-  
63 sion.

64 As a philosophy of mathematics, social constructivism, as defined by Ernest (1991),  
65 views mathematics as a social construction. It is based on conventionalism, which  
66 acknowledges that "human language, rules and agreement play a role in establishing  
67 and justifying the truths of mathematics" (p. 42). Ernest gives three grounds for this  
68 philosophy. The first is that linguistic knowledge, conventions and rules form the  
69 basis for mathematical knowledge. The second is that interpersonal social processes  
70 are needed to turn an individual's subjective mathematical knowledge into accepted  
71 objective knowledge. The last is that objectivity is understood to be social. A key  
72 part of what separates social constructivism from other philosophies of mathematics  
73 is that it takes into account the interplay between subjective and objective knowl-  
74 edge. When a discovery is made by an individual, this subjective knowledge later  
75 becomes knowledge accepted by the community—thus becoming objective. Then,  
76 as this knowledge is further spread to others, they internalize it and it becomes sub-  
77 jective again.

78 However the philosophy is not without its critics. Gold (1999) raises several  
79 objections. The first is that this philosophy fails to account for the usefulness of  
80 mathematics in the world. Social constructivism does fine when explaining how  
81 mathematics can be created to solve practical problems. However, it does nothing  
82 to explain mathematics created long before application. Social constructivism also  
83 fails to account for cases like that of Ramanujan, who developed his results through  
84 interaction with mathematical objects and not a mathematical community. Gold's  
85 main critique, however, is the failure of social constructivism to distinguish between  
86 mathematical knowledge and mathematics itself. Mathematical knowledge is what  
87 is socially created and/or discovered. She repeatedly draws on physics as an illus-  
88 tration. "(P)hysical objects either are or are not made up of atoms, and it is not the  
89 community of physicists that makes that true or false" (Gold 1999, p. 377). While  
90 our knowledge of something may change over time, the reality of it does not. If  
91 mathematics is a human creation, can the same not be said for the quarks? Social  
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93 constructivists would point to the fallibility of proofs as evidence that mathematics  
94 is a social construct and therefore lacks certainty. If the verification of mathemati-  
95 cal facts can turn out to be false, then mathematical facts are subject to question as  
96 well. Gold points out, though, that proofs are among the activities that concern hu-  
97 man knowledge. As such, they are subject to revision, as are theories in the physical  
98 sciences that mean to explain some physical phenomenon. The revision of explan-  
99 atory theory, however, does not change the physical phenomenon. Hence, the social  
100 constructivist philosophy of mathematics is not a philosophy of mathematics edu-  
101 cation per se, but it does have educational implications. Social constructivism as a  
102 philosophy of mathematics can serve as a basis for developing a theory of learning,  
103 such as constructivism (Sriraman and English, this volume).

104 What implications if any does such a philosophy have for the necessity, teaching  
105 and learning of proof. Ontologically speaking, social constructivism would say that  
106 a mathematical proof becomes one when it is accepted by the community, and given  
107 the status that "result  $x, y, z$ , etc. exist". In other words, the burden of mathematical  
108 proof is that it must convince others. The largest implication this has on mathematics  
109 education is that students need to learn that this is what a proof is meant to do (versus  
110 the idea that proof is a logical deduction from known facts). The philosophy of social  
111 constructivism also has an epistemological implication for the teaching and learning  
112 of proof. This is the realization that mathematical proof has its origin in human  
113 activity and is therefore is in a sense fallible and dynamic. They also need to be  
114 made aware that the burden of proof has changed at different times, depending on the  
115 rigor demanded by certain mathematical communities. In this way, they will realize  
116 that they need be sensitive to what is considered proof in their community. While a  
117 mathematician needs to be aware of what will constitute a proof within his or her  
118 community, students need to be taught it. Mathematicians make use and are aware  
119 of the methods that are recognized as valid in the community and students need to be  
120 taught those methods. As a philosophy of mathematics, social constructivism aims  
121 to describe what mathematics truly is and what is done by those in the field. On the  
122 other hand, as a philosophy of mathematics education, its aim is to train students in  
123 a way that is reflective of this view of mathematics as a whole.

124 Simon Goodchild points out in his commentary on Ernest's chapter *Reflections*  
125 *on theories of learning*, the words philosophies and theories often get used inter-  
126 changeably by him, when the former is what is intended since the latter bears a much  
127 higher burden of testability in order to garner acceptance. Ernest is well aware of  
128 this distinction as his chapter unfolds into the different strains of constructivism and  
129 their relevance for learning. The second commentary to the chapter, is Ernest's own  
130 reflections to his previous chapter *Reflections on theories of learning*. This lends  
131 the metacognitive spin that Simon Goodchild lamented was lacking in the original  
132 chapter, albeit this meta-cognition is being engaged in strictly at a theoretical level!

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# Reflections on Theories of Learning

Paul Ernest

**Prelude** Four philosophies of learning are contrasted, namely ‘simple’ constructivism, radical constructivism, enactivism and social constructivism. Their underlying explanatory metaphors and some of their strengths and weaknesses are contrasted, as well as their implications for teaching and research. However, it is made clear that none of these ‘implications’ is incompatible with any of the learning philosophies, even if they sit more comfortably with one of them.

## 1 Construction

Constructivism has been a leading if not the dominant theory or philosophy of learning in the mathematics education research community ever since the heated controversy in the 1987 Montreal PME conference. What made constructivism such a hot issue was not just what it claims about learning. Rather it is the epistemological implications that follow from it. As one of the leading exponents of constructivism said “To introduce epistemological considerations into a discussion of education has always been dynamite” (Glaserfeld 1983: 41).

But constructivism does not represent a single school of thought, as there are several versions and varieties, some diametrically opposed to others. What binds many of the various forms of constructivism together is the metaphor of construction from carpentry or architecture. This metaphor is about the building up of structures from pre-existing pieces, possibly specially shaped for the task. In its individualistic form the metaphor describes understanding as the building of mental structures, and the term ‘restructuring’, often used as a synonym for ‘accommodation’ or ‘conceptual change’ in cognitivist theory, contains this metaphor. What the metaphor need not mean in most versions of constructivism is that understanding is built up from received pieces of knowledge. The process is recursive (Kieren and Pirie 1991), and so the building blocks of understanding are themselves the product of previous acts of construction. Thus the distinction between the structure and content of understanding can only be relative in constructivism. Previously built structures become the content in subsequent constructions. Meanings, structures and knowledge are emergent.

The metaphor of construction is implicit in the first principle of constructivism as expressed by von Glaserfeld (1989: 182): “knowledge is not passively received but actively built up by the cognizing subject”. One can term ‘simple constructivism’

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Throughout this paper for brevity what I refer to as learning theories might more accurately be termed philosophies of learning. Some might argue that these ‘theories’ are not specific or testable (i.e., falsifiable) enough to deserve this title.

47 those positions based on this principle alone. Basic as this simple form might be, it  
48 represents a very significant step forward from naive empiricism or classical behav-  
49 iourism. For it recognizes that knowing is active, that it is individual and personal,  
50 and that it is based on previously constructed knowledge. Just getting student teach-  
51 ers to realize this, by reflecting on ‘child methods’ in mathematics or alternative  
52 conceptions in science, say, represents a significant step forward from the naive  
53 transmission view of teaching and passive-reception view of learning many student  
54 teachers arrive with. Unfortunately a passive-reception view of learning is not dead  
55 among professionals or administrators in education. Many government driven cur-  
56 riculum reforms, in Britain and elsewhere, assume that the central powers can sim-  
57 ply transmit their plans and structures to teachers who will passively absorb and then  
58 implement them in ‘delivering the curriculum’. Such conceptions and strategies are  
59 deeply embedded in the public consciousness, although it may be no accident that  
60 they also serve authoritarian powers (Ernest 1991). Freire (1972) is critical about  
61 the ‘banking’ model of learning in which inert items of knowledge are passed over  
62 to learners who have to absorb them, thus becoming passive receptors rather than  
63 epistemologically and politically empowered social agents.

64 What has been termed ‘simple constructivism’ can be applied as a descriptor  
65 to some extent to neo-behaviourist and cognitive science learning theories. These  
66 should not be dismissed too lightly. The models of cognition in the work of Ausubel,  
67 Gagné and others, are subtle and complex. As long ago as 1968 Ausubel wrote that  
68 “The most important single factor influencing learning is what the learner already  
69 knows. Ascertain this, and teach him accordingly.” (Ausubel 1968: 18). This is a  
70 principle shared by most forms of constructivism, asserting that pre-existing knowl-  
71 edge and understandings are the basis for virtually all subsequent learning.

72 In discussing the metaphor of construction, there is an important distinction to be  
73 drawn between individual and social construction. The cognitivist and constructivist  
74 accounts I have referred to are based on the metaphor applied within the individual,  
75 in which a learner constructs their knowledge and understanding internally based on  
76 their personal interpretation of their experiences and their pre-existing knowledge.  
77 This account could be extended to include the individual construction of various  
78 affective responses, including attitudes, beliefs and values, and even learners’ entire  
79 personalities, for some versions of constructivism. Nonetheless, this constitutes an  
80 individualistic form of construction. In contrast, another use of the metaphor lies in  
81 social construction, in which the learning and knowledge construction takes place  
82 in the social arena, in the ‘space between people’, even if its end products are ap-  
83 propriated and internalized by those persons individually.

84 One of the key differences between different forms of constructivism, and learn-  
85 ing theories in general, is whether it is assumed that absolute knowledge is attainable  
86 or not. Simple constructivism and most cognitive science theories of learning accept  
87 that true representations of the empirical and experiential worlds are possible. This  
88 is not the case with radical constructivism.  
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## 2 Radical Constructivism

Although it originates with Piaget, and is partly anticipated by Vico, Kant and others, in its modern form radical constructivism has been most fully worked out in epistemological terms by von Glasersfeld and colleagues. Definitionally, radical constructivism is based on both the first and second of von Glasersfeld's principles, that latter of which states that "the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality." (von Glasersfeld 1989: 182). Consequently, "From an explorer who is condemned to seek 'structural properties' of an inaccessible reality, the experiencing organism now turns into a builder of cognitive structures intended to solve such problems as the organism perceives or conceives." (Glasersfeld 1983: 50).

This suggests an underlying metaphor for the mind or cognizing subject in radical constructivism is that of an organism undergoing evolution, patterned after Darwin's theory, with its central concept of the 'survival of the fit'. This is indicated in Piaget's notion of adaptation to the environment, and his explicit discussion of cognitive evolution, such as in Piaget (1972). According to the evolutionary metaphor the cognizing subject is a creature with sensory inputs, furnishing data that is interpreted (or rather constructed) through the lenses of its cognitive structures; it comprises also a collection of those structures all the while being adapted; and a means of acting on the outside world. The cognizing subject generates cognitive schemas to guide actions and represent its experiences. These are tested according to how well they 'fit' the world of its experience. Those schemas that 'fit' are tentatively adopted and retained as guides to action. Cognition depends on an underlying feed-back loop.

Thus on the one hand, there is an analogy between the evolution and survival of the fitter of the schemas in the mind of the cognizing subject and the whole of biological evolution of species. Schemas evolve, and through adaptation come to better fit the subject's experienced world. They also split and branch out, and perhaps some lines become extinct. On the other hand, the organism itself and as a whole, is adapting to the world of its experiences, largely through the adaptation of its schemas.

A widespread criticism of radical constructivism and indeed of other learning philosophies based on the individual conception of construction is that the account of the cognizing subject emphasizes its individuality, its separateness, and its primarily cognitive representations of its experiences. Its representations of the world and of other human beings are personal and idiosyncratic. Indeed, the construal of other persons is driven by whatever representations best fit the cognizing subject's needs and purposes. None of this is refutable. But such a view makes it hard to establish a social basis for interpersonal communication, for shared feelings and concerns, let alone for shared values. By being based on the underlying evolutionary metaphor for the mind there is a danger that interpersonal relations are seen as nothing but competitive, a version of the 'law of the jungle'. After all, this is but another way of phrasing 'the survival of the fit'. Yet society and its functions, in particular education, depend on articulated and shared sets of concerns and values.

139 Values which are most evidently subscribed to by radical constructivists themselves.  
140 Thus the paradigm needs to accommodate these issues by balancing knowing with  
141 feeling, and acknowledging that all humans start as part of another being, not sepa-  
142 rate.

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### 146 3 Enactivism

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148 Since the 1990s, following the publication of the influential work *The Embodied*  
149 *Mind* (Varela *et al.* 1991), enactivism has become increasingly popular as a theory  
150 of learning among mathematics education researchers. One of the central ideas is  
151 that of autopoiesis. This is the property of complex dynamic systems of spontaneous  
152 self-organization, based on feedback loops and growth in response to this feedback.  
153 Enactivism is a theory of cognition as “the enactment of a world and a mind on the  
154 basis of a history of the variety of actions that a being in the world performs” (Varela  
155 *et al.* 1991: 9). In other words, the individual knower is not simply an observer of the  
156 world but is bodily embedded in the world and is shaped both cognitively and as a  
157 whole physical organism by her interaction with the world. “Enactivism as a theory  
158 of cognition acknowledges the importance of the individual in the construction of a  
159 lived world, but emphasizes that the structure of the individual coemerges with this  
160 world in the course of, and as a requirement for, the continuing inter-action of the  
161 individual and the situation.” (Reid *et al.* 2000: I-10)

162 Another source of enactivism is the theory of the bodily basis of thought via  
163 the role of metaphors, drawing on the work of George Lakoff and Mark Johnson  
164 (Lakoff and Johnson 1980; Johnson 1987). This proposes that all human under-  
165 standing, including meaning, imagination, and reason is based on schemes of bodily  
166 movement and its perception (“image schemata”, Johnson 1987: xiv). These are ex-  
167 tended via metaphor (“metaphorical projection” *op. cit.* xv), providing the basis for  
168 all human understanding, thought and communication. Recently Lakoff and Núñez  
169 (2000) even developed the ideas embodied metaphors so as to offer an account of  
170 the discipline of mathematics.

171 These bases of enactivism provide a rich and powerful explanatory theory for  
172 learning and being. It is being applied in a number of research studies, especially  
173 by Canadian researchers in mathematics education. However, I want to suggest that  
174 it is not so very different from Piaget’s epistemology and learning theory and the  
175 radical constructivism to which it gave birth. Indeed Piaget and Bruner are the only  
176 two psychologists that are charted in the territory of enactivism in the map provided  
177 by Varela *et al.* (1991: 7). After all, Piaget’s central mechanism of equilibration (the  
178 achievement of balance within the knower in response to perturbations) is based on  
179 a similar biological model of a being in interaction with its environment.

180 Reid (1996: 2) claims that “There is an important distinction to be made, how-  
181 ever, with some constructivist perspectives. It is not a matter of an individual having  
182 a cognitive structure, which determines how the individual can think, or of there  
183 being conceptual structures which determine what new concepts can develop. The  
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185 organism as a whole *is* its continually changing structure which determines its own  
186 actions on itself and its world. This holistic vision of the cognitive entity is central”.

187 However, this seems to me to be a matter of emphasis rather than a major shift.  
188 Perhaps more significant is the emphasis of enactivism on metaphor, which does  
189 not figure so explicitly in Piaget (or radical constructivist accounts). Piaget does  
190 emphasise ‘reflective abstraction’ as a mechanism whereby concepts and schemas  
191 are abstracted and generalized, and metaphorical thinking might be seen as one of  
192 the modes of this.

193 The assumption that bodily metaphors and their enactivist/imagistic basis provide  
194 the foundations for subsequently more developed concepts is not without its  
195 weaknesses. “Bachelard regards the common-sense mind’s reliance on images as  
196 a breeding ground for epistemological obstacles ... [these] are often not explicitly  
197 formulated by those they constrain but rather operate at the level of implicit  
198 assumptions or cognitive or perceptual habits.” (Gutting 1990: 135). Thus naïve notions  
199 like those derived from bodily metaphors may underpin misconceptions, such  
200 as the quasi-Aristotelian notions that Alternative Frameworks researchers in science  
201 education have documented extensively (Pfundt and Duit 1991)

202 What both enactivism and radical constructivism appear to share is the subordi-  
203 nation of the social or the interpersonal dimension, and indeed the existence of other  
204 persons to constructions and perceived regularities in the experienced environment.  
205 The knowers’ own body might be a given, albeit emergent, but other persons’ bodies  
206 and overall beings are not. Ironically, language, which is the primary seat of  
207 metaphor, is the quintessential social construction. But language, like other persons,  
208 seems to be removed and exterior to the primary sources of knowledge of the enac-  
209 tive self in these perspectives.

## 211 4 Social Constructivism

212  
213 There are a variety of social constructivist positions, but for simplicity I shall  
214 treat them as one, based on the seminal work of Vygotsky. Social constructivism  
215 regards individual learners and the realm of the social as indissolubly intercon-  
216 nected. Human beings are formed through their interactions with each other as  
217 well as by their individual processes. Thus there is no underlying model for  
218 the socially isolated individual mind. Instead, the underlying metaphor is di-  
219 alogical or ‘persons-in-conversation’, comprising socially embedded persons in  
220 meaningful linguistic and extra-linguistic interaction and dialogue (Harré 1989;  
221 Ernest 1998). However, this metaphor for conversation is not the bourgeois chat-  
222 ter of the dining or breakfast-table (e.g., Holmes 1873) no matter how profound  
223 the discussion. Rather it is like the directed talk of workmen accomplishing some  
224 shared task, such as “bring me a slab” (Wittgenstein 1953: 8). In Wittgensteinian  
225 terms these social contexts are shared ‘forms-of-life’ and located in them, shared  
226 ‘language-games’.

227 From this perspective, mind is viewed as social and conversational, because first  
228 of all, individual thinking of any complexity originates with and is formed by inter-  
229 nalised conversation. Second, all subsequent individual thinking is structured and  
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		<i>Social Location</i>		
		Individual		Collective
<i>Ownership</i>	Public	Individual's public utilization of sign to express personal meanings	Conventionalisation →	Conventionalized and socially negotiated sign use (via critical response & acceptance)
	Private	Individual's development of personal meanings for sign and its use	← Transformation	Individual's own unreflective response to and imitative use of new sign utterance
		Publication ↑		↓ Appropriation

**Fig. 1** Model of sign appropriation and use

natured by this origin; and third, some mental functioning is collective (e.g., group problem solving, sign-based learning). These assumptions stem from the Vygotskian developmental account of the origins of language in the individual as something that is internalised and appropriated from social functioning. Vygotsky (1978) describes how the spontaneous concepts that children form through their perceptions merge with the more 'scientific' concepts that are linguistically mediated that are acquired through such social activity. Through play the basic semiotic fraction of signifier/signified begins to become a powerful factor in the social (and hence personal) construction of meaning. Thus for Vygotsky bodily activities result in spontaneous concepts but these only become abstracted (e.g., into metaphors) through symbolic mediation, i.e., the acquisition, structuring and use of language and other semiotic systems.

Conversation offers a powerful way of accounting for both mind and learning. A Vygotskian theory of the development of mind, personal identity, language and knowledge, can be represented in a cycle of appropriation, transformation, publication, conventionalisation. This shows different aspects of the use of signs, understood here within a semiotic perspective that sees signifiers as any publicly presented or uttered representation or text and signifieds as meanings that are often woven indissolubly into the social and cultural fabric through the roles and patterns of use of the signs. Beyond this, the component activities of sign reception and production involved in language games are woven together within the larger epistemological unit of conversation (Ernest 1998; Harré and Gillett 1994; Shotter 1993). A schematic model of the way in which these two activities are mutually shaping is shown in Fig. 1.

Figure 1 illustrates how signs become appropriated by an individual through experiencing their public use. In the first instance this leads to the learner's own unreflective response to and imitative use of signs, based on the perceived regularity (rule-based) and connectivity of use within the discursive practice. After a series of such uses and public sign utterances, in which the whole cycle may be brought into play in miniature, a nexus of implicit rules and associations with actions and signs for the sign are learned. In this process the individual develops personal meanings for the sign and its use, transforming it into something that is individually and privately owned. The individual is now able to utilize the sign in autonomous

277 conversational acts, the publication stage, which can vary in scope from relatively  
 278 spontaneous utterances, to the construction of extended texts. These productions are  
 279 subject to the process of conventionalisation in which an individual's public sign  
 280 utterances offered in various modes of conversation are subjected to attention and  
 281 response which can be critique, negotiation, reformulation or acceptance.

282 The process of conventionalisation takes place at the visible centre of the Zone  
 283 of Proximal Development (Vygotsky 1978) in which the learner experiences and  
 284 is guided by interventions over public sign use. The processes of appropriation  
 285 and publication are boundary operations between the public and private domains  
 286 in which the learner participates in the communicative activity of sign reception and  
 287 production. In the private domain the learner transforms collective signs into indi-  
 288 vidual ones through the production of meaning, and is thus a pivotal location for  
 289 learning. In fact the ZPD might be said to encompass all four quadrants in which  
 290 sign use is being learned.

291 The model thus describes an overall process in which both individual and pri-  
 292 vate meanings and collective and public expressions are mutually shaped through  
 293 conversation. Knowledge and the meaning of the full range of signs texts and other  
 294 cultural forms of representation is distributed over all four quadrants of the model.  
 295 The model represents a micro view of learning and of knowledge production. How-  
 296 ever it is limited in its focus in that it does not accommodate the issues of power and  
 297 the larger social structures through which knowledge, power and economics are mu-  
 298 tually constitutive and circulate. For this a further analysis of knowledge is required,  
 299 beyond what can be given here.

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## 302 **5 Implications for Educational Practice**

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304 Ultimately, the import of a learning theory concerns its implications for practice,  
 305 both pedagogically, in the teaching (and learning) of mathematics, and in the prac-  
 306 tice of conducting educational research. However, in my view, there is little in any  
 307 pedagogy that is either wholly necessitated or wholly ruled out by the other elements  
 308 of a learning theory. Similarly, learning theories do not imply particular research  
 309 approaches. Nevertheless, certain emphases are foregrounded by different learning  
 310 theories, even if they are not logical consequences of them.

311 Simple constructivism suggests the need and value for:

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313 (1) sensitivity towards and attentiveness to the learner's previous learning and con-  
 314 structions,

315 (2) identification of learner errors and misconceptions and the use of diagnostic  
 316 teaching and cognitive conflict techniques in attempting to overcome them.

317 Radical constructivism suggests attention to:

318

319 (3) learner perceptions as a whole, i.e., of their overall experiential world,

320 (4) the problematic nature of mathematical knowledge as a whole, not just the  
 321 learner's subjective knowledge, as well as the fragility of all research method-  
 322 ologies.

Enactivism suggests that we attend to:

- (5) bodily movements and learning, including the gestures that people make,
- (6) the role of root metaphors as the basal grounds of learners' meanings and understanding.

Social constructivism places emphasis on:

- (7) the importance of all aspects of the social context and of interpersonal relations, especially teacher-learner and learner-learner interactions in learning situations including negotiation, collaboration and discussion,
- (8) the role of language, texts and semiosis in the teaching and learning of mathematics.

However, each one of these eight focuses in the teaching and learning of mathematics could legitimately be attended to by teachers drawing on any of the learning theories for their pedagogy, or by researchers employing one of the learning theories as their underlying structuring framework.

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# Commentary 1 on Reflections on Theories of Learning by Paul Ernest

Simon Goodchild

Paul Ernest is an internationally recognised authority on the philosophy of social-constructivism particularly in the context of mathematics education. He has published widely on the issue, perhaps his two best known and widely cited works are ‘The Philosophy of Mathematics Education’ (Ernest 1991), and ‘Social Constructivist as a Philosophy of Mathematics’ (Ernest 1998). As one engages with this short paper it is evident that one is in the company of a ‘master’ of the topic. It is quite remarkable how within the space of about 4000 words he manages to produce an erudite and informative account of 4 related theories of learning, and outline some of their implications for teaching.

Before briefly summarising the paper it is worth drawing attention to a point that Ernest explains in an end note. The title refers to ‘theories’ of learning but in the first line of the abstract this is transformed into ‘philosophies’ of learning. In the end note Ernest explains that the “‘theories’ are not specific or testable (i.e. falsifiable) enough to deserve’ the title ‘theories’ (p. 7). This is an important observation and one that is not often made—the theories of learning, upon which much of the research in the field of mathematics education is founded, are untested mainly because in many respects they are not testable in the ‘traditional’ scientific sense. Ernest does not substantiate this assertion, and I will not attempt the task here. Despite this observation Ernest continues the paper using the word ‘theories’, he explains, for the sake of ‘brevity’.

The paper focuses on four major constructive models of cognition although in the detail reference is made briefly to other learning theories which appear, in Ernest’s opinion to be close to constructivism. Ernest considers simple constructivism, radical constructivism, Enactivism, and Social Constructivism. It might be worth noting that ‘simple constructivism’ has also been described as ‘weak constructivism’ (Lerman 1989). Ernest sets out by explaining the basic metaphor of constructivism and provides sufficient detail of the introduction and development of this learning theory within mathematics education, including reference to a significant international conference in 1987, which would allow the interested reader to explore much further. However as Ernest points out, constructivist ideas have been around since the time of the philosophers Vico (1668–1744), and Kant (1724–1804), and then identifies Piaget (1896–1980) as the one from whom constructivist theories of learning ‘originate’ from about the middle of the twentieth century.

Simple constructivism and radical constructivism are based on a common metaphor of ‘construction’. Learning is about ‘conceptual change’ where ‘the building blocks of understanding are themselves the product of previous acts of construc-

47 tion' (p. 3). These two versions of constructivism are distinguished in that the former  
48 'simple constructivism and most cognitive science theories of learning accept  
49 that true representations of the empirical and experiential world are possible' (p. 4).  
50 However in radical constructivism it is argued that the best that can be achieved  
51 is for mental representations to 'fit' experience, because there can be no grounds  
52 for any assurance that representations ever achieve a perfect match. The function of  
53 cognition is to achieve the viability of mental representations. Ernest observes that  
54 a 'widespread criticism of . . . philosophies based on the individual construction is  
55 that the account of the cognizing subject emphasizes its individuality' . . . and 'it is  
56 hard to establish a social basis for interpersonal communication, for shared feelings  
57 and concerns, let alone for shared values.' This is, indeed an accurate representation.  
58 However, in the literature of, particularly radical constructivism this critique  
59 is addressed and those working from within this philosophical perspective would  
60 question the validity of the claim (see for example the chapters in the comprehensive  
61 work 'Constructivism in Education' edited by Steffe and Gale 1995).

62 Ernest could have drawn attention to other often cited criticism of constructivism,  
63 such as the problem of 'bootstrapping' which refers to how the construction process  
64 begins, what are the initial building blocks of cognition, and what is the mechanism  
65 that enables construction—is this learned or innate? Given more space it is reasonable  
66 to believe that Ernest would have made mention of other critiques. By drawing  
67 attention to the realm of the social in particular provides a rationale for looking at  
68 further developments of constructivism and thus to consider enactivism and social  
69 constructivism.

70 Enactivism is based on a biological model; more specifically, cognition is seen as  
71 a biological process. Ernest explains 'one of the central ideas (of enactivism) is that  
72 of autopoiesis. This is the property of complex dynamic systems of spontaneous self-  
73 organization, based on feedback loops and growth in response to this feedback . . .  
74 the individual knower is not simply an observer of the world but is bodily embedded  
75 in the world and is shaped both cognitively and as a whole physical organism by her  
76 interaction with the world' (p. 4). Ernest briefly examines the major features of  
77 enactivism and argues that it does not represent a major shift from the other forms  
78 of constructivism already discussed, more a matter of emphasis. He then moves on  
79 to inform of one criticism that draws attention to an argued weakness entailed in  
80 establishing learning theory on simple metaphors. Briefly the argument is that the  
81 metaphors can as often constrain thinking as much as enable it.

82 Given the conceptual proximity of the three theories discussed so far, it is argued  
83 that still insufficient attention is paid to the social, and thus the ground for setting  
84 out the case for social constructivism is laid. Given Ernest's reputation as a leading  
85 figure in the development of social constructivism as a philosophy of mathematics  
86 education it is perhaps reasonable to suggest that the treatment of the three theories  
87 has been organised to lead to the inevitability of social constructivism. This is not to  
88 suggest that Ernest is being disingenuous. Rather, it is reasonable to assume that his  
89 espousal of social constructivism is based on a rational appreciation of competing  
90 theories and he has attempted to lead the reader through his reasoning processes.

91 The underlying metaphor of social constructivism according to Ernest is 'dialogical  
92 or "persons-in-conversation"', comprising socially embedded persons in mean-

93 ingful linguistic and extra-linguistic interaction and dialogue' (p. 5). Ernest then  
94 provides a brief but informative overview of social constructivism, as one might  
95 anticipate referring to the seminal work of Vygotsky, and also to other important  
96 contributions by, for example, Wittgenstein, Harré, and Shotter. Ernest provides a  
97 model of the way cognitive processes are woven together 'within the larger episte-  
98 mological unit of conversation' (p. 5), which he helpfully explains using both words  
99 and graphics. Ernest admits that 'the model represents a micro view of learning and  
100 knowledge production . . . it does not accommodate issues of power and larger social  
101 structures through which knowledge. Power and economics are mutually constitu-  
102 tive and circulate' (p. 6).

103 In that Ernest sets out to contrast four theories of learning' it is understandable  
104 that his account of the four theories ends at this point. However, the scholarly debate  
105 about learning theories does not end at this point. Lerman, for example has argued  
106 that social constructivism is 'incoherent' (Lerman 1996). The exchange of papers  
107 published in the Journal for Research in Mathematics Education provoked by Ler-  
108 man illustrate the way that scholars can talk past each other and can not engage with  
109 the arguments because they are based on different fundamental premises (Kieren  
110 2000; Lerman 2000; Steffe and Thompson 2000). The fundamental differences are  
111 set out by Roth and Lee (2007) who explain the 'dialectical nature of consciousness'  
112 (Roth and Lee 2007, p. 195) that underpins the socio-cultural theories of cognition.  
113 The fundamental divergence then between constructivist theories and socio-cultural  
114 theories arises from the dualistic individual self-other basis of constructivism and  
115 the dialectics of Vygotsky. It is not clear from Ernest's paper where he would place  
116 his version of social constructivism. As a version of constructivism one assumes  
117 it is based on dualistic notions of self-other, however the notion of conversation is  
118 predicated on the presence of an 'other' and thus it appears to be dialectical.

119 These constructivist theories of learning do not, of themselves, entail a theory of  
120 teaching, but as Ernest observes they have implications for teaching and this is one  
121 reason why learning theories are important. For each theory Ernest provides two  
122 implications for teaching, thus altogether eight implications. These include—very  
123 briefly summarised—for simple constructivism: attention to prior learning, attention  
124 to misconceptions and errors; for radical constructivism: attention to learner's per-  
125 ceptions of their experiential world, the problematic nature of knowledge; for enac-  
126 tivism: giving attention to bodily movements and learning, and the role of metaphor;  
127 for social constructivism: all aspects of the social context, and the role of language  
128 text and signs and signals. It may be surprising that no mention is made of the role  
129 of metacognition, given the attention that this has received in research in mathemat-  
130 ics education, and readers might wonder about other issues dear to them. However,  
131 it must be recognised that Ernest has made his choices and limited himself to just  
132 two issues for each of the theories considered. The key point that Ernest makes in  
133 concluding his article, and I think it is also fair to allow him the final word in this  
134 commentary:

135 each one of these eight focuses in the teaching and learning of mathematics could legiti-  
136 mately be attended to by teachers drawing on any of the learning theories for their pedagogy,  
137 or by researchers employing one of the learning theories as their underlying structuring  
138 framework.

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# Commentary 2 on Reflections on Theories of Learning

Paul Ernest

One of the great clashes of ideas of our times is that between psychology and sociology. Sociologists accuse psychologists of being narrowly technical, supporters of what the critical theorists term instrumental rationality. Psychologists have also been stereotyped as apolitical and closed minded about social and political issues and social/political influences on human life in general and on learning in particular. In return, psychologists accuse sociologists of sacrificing scientific truth and accuracy of detail for broad politically motivated generalizations that do not help people with their interior lives and their learning. Ironically, both sets of accusations are both true at times and false at others. Because both psychology and sociology are broad areas of thought housing many ideas and schools. Sociology can be mechanistic, on the one hand, focusing on structural mechanisms that leave the individual relatively without agency or an internal life. On the other hand sociological explanations can be rich and multi-faceted exploring individual agency and power in the construction of knowledge and institutions.

Similarly psychology encompasses a broad range of schools from the behaviourism and experimental psychology, at one extreme, to discursive psychology and socio-cultural theory, at the other. Somewhere in between these extremes lies the range of constructivisms I explored with respect to their role in the learning of mathematics. I contrasted four positions referred to under the titles of simple constructivism, radical constructivism, enactivism and social constructivism. These four learning theories, or learning philosophies as they should more accurately be termed,<sup>1</sup> constitute a sequence in this order, and in accord with this order they become increasingly radicalized in terms of their epistemology, become more embodied in terms of the learner's physicality and context, and shift from a primarily individual focus to the inclusion of the social. Although this characterisation in broad brush strokes is more or less correct it is not fully accurate not least because radical constructivism and enactivism do not differ much in these dimensions. Both theories acknowledge the embodied nature of the learner, but prioritize the individual over the social.

However, as this discussion of the breadth of psychology shows, my choice of theories to comment on in Ernest (2006) represents a limited selection of learning theories that are current in psychology. They all fall within what Wegerif

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<sup>1</sup>In my original paper I argue that learning theories are better termed learning philosophies because they are not specific or testable enough to be theories, strictly speaking. Nevertheless, I shall follow common usage in using the description 'learning theory'.

(2002) terms the cognitivist/ constructivist orientation to learning. The one exception is that of social constructivism, which has two formulations, Piagetian/radical constructivist and Vygotskian (Ernest 1994). The former fits within the cognitivist/constructivist orientation, whereas the latter sits closer to Wegerif's 'participatory' orientation to learning.

Wegerif (2002) contrasts four orientation to learning, as he terms them, thus sidestepping the theory/philosophy issue I noted above.<sup>2</sup> These are the Behaviourist, Cognitivist/Constructivist, Humanist, and Participatory orientations. For each of these orientations he considers the best known learning theorists who are founders or adherents of the position, the view of the learning process from this perspective, the locus of learning, the view of transfer, the purpose of education, and the educator's role. These are summarized in Table 1.

The Behaviourist orientation is an important one to include for two reasons. First it is historically important. It was a if not *the* leading theory of learning for at least half of the twentieth century. Secondly, it represents one of the extreme anchor points for the range of psychological theories. It constitutes one end of the spectrum that goes from the hard-nosed individualistic and scientific through to, at the other extreme, fully social theories of learning represented here by the participatory orientation. Although much pilloried and used as a 'straw person' against which to argue for constructivist and other theories of learning, modern neo-behaviourist theories of learning as formulated by such scholars as Gagne and Ausubel in fact incorporate many of the insights of cognitivist/constructivist theories.

It is also an irony that behaviourism shares one of its main characteristics with the latest participatory or socio-cultural theories, thus confirming the popular (and only partly humorous suggestion) that two the two extreme poles of any continuum meet up 'around the back', as it is claimed do extreme left and right wing political ideologies. For both extremal learning theories reject the characterization of learning as an 'interior' activity that takes place within the learner. Behaviourism sought to be a scientific theory focussing on objectively observable as opposed to subjective phenomena. Socio-cultural theory likewise focuses on social behaviour between persons. Socio-cultural theory rejects the model of learning in terms of the internal accumulation of knowledge, described by Freire (1972) as the 'banking model' and by Sfard (1998) as the acquisition metaphor. Instead socio-cultural theory focuses on participation in social practices, the learning of social roles and behaviours. The emphasis is on tacit knowledge learned by osmosis through immersion in and participation in social practices, that is in socially organised productive activities of one sort or another.

Wegerif's characterization of Cognitivist/Constructivist orientations, the second column in Table 1, fits fairly well with my finer grained analysis of varieties of constructivism (Ernest 2006) so I will leave these theories out of my discussion for now.

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<sup>2</sup>Wegerif acknowledges his indebtedness to Merriam and Caffarella (1991) in making these distinctions.

**Table 1** Four orientations to learning (from Wegerif 2002: p. 10)

Aspect	Behaviourist	Cognitivist/ Constructivist	Humanist	Participatory
Learning theorists	Thorndike, Pavlov, Watson, Tolman, Skinner, Suppes	Piaget, Ausubel, Bruner, Papert	Maslow, Rogers	Lave, Wenger, Cole, Wertsch, Engestrom
View of the learning process	Change in behaviour	Internal mental process including insight, information processing, memory, perception	A personal act to fulfil potential e	Interaction/ observation in social contexts. Movement from the periphery to the centre of a community of practice
Locus of learning	Stimuli in external environment	Internal cognitive structuring	Affective and cognitive needs	Learning is in relationship between people and environment
View of transfer	Common elements shared by different contexts	Over-arching general principles	Changes in self-identity as a learner	Transfer problematic
Purpose in education	Produce behavioural change in desired direction	Develop capacity and skills to learn better	Become self-actualized, autonomous	Full participation in communities of practice and utilization of resources
Educator's role	Arranges environment to elicit desired response	Structures content of learning activity	Facilitates development of the whole person	Works to establish communities of practice in which conversation and participation can occur

Wegerif's third orientation summarized in Table 1 is the humanistic orientation. Although this has not figured largely in mathematics education research as such, it incorporates some useful emphases worth highlighting here. As is well known, it follows on from the humanistic psychology tradition founded by scholars like Abraham Maslow and Carl Rogers. It focuses on the whole person, rather than on isolated cognitive processes and mechanisms. Learning is seen as a personal act to fulfil an individual's own potential and thus to meet their affective and cognitive needs in the round. It focuses on changes in self-identity as a learner. Identity is a theme that is becoming increasingly central in mathematics education research even though its

139 roots in the humanistic tradition are rarely acknowledged.<sup>3</sup> This perspective aims  
140 to enable students and indeed all persons to become self-actualized, autonomous  
141 human beings, thus facilitating the development of the whole person, that is, their  
142 overall fulfilment. There is still much to be learned from this perspective which has  
143 the unique emphasis of treating persons first and foremost as human beings. Such  
144 an emphasis inevitably brings with it a moral and ethical dimension, something that  
145 is regarded as irrelevant or secondary by most of the other learning theories.

146 The fourth and last of Wegerif's orientations he terms participatory. More com-  
147 monly in mathematics education research this is termed socio-cultural theory, al-  
148 though in equating these titles I may be committing the same error that I wish to  
149 criticize about this orientation. A number of major contributing modern thinkers  
150 are listed including Lave, Wenger, Cole, Wertsch, and Engestrom. Jean Lave is an  
151 anthropologist, who collaborated with her student Etienne Wenger to develop an  
152 account of situated learning and apprenticeship described as legitimate peripheral  
153 participation (Lave and Wenger 1991). Although much lauded, this account paid  
154 scant regard to the role of explicit knowledge in education and learning. Its main  
155 theoretical foundation lies in Vygotsky's Activity Theory. Wenger (1998) elaborated  
156 and extended these ideas in his treatment of Communities of Practice, focussing on  
157 sub-themes of learning, meaning and identity. This book also caused a great stir in  
158 the mathematics education research community and beyond, including studies of  
159 information and communication technology and learning, and learning in business  
160 communities. Although seductively rich in new concepts and models, as has been  
161 said in reviews of the work (e.g., Ernest 2002), as yet it lacks an adequate theoret-  
162 ical grounding. Vygotsky is no longer the central underpinning theorist, but he lacks  
163 a coherent replacement Wenger is very eclectic in drawing from many disciplines,  
164 but ultimately this leads to a lack of a solid foundation, something that is shared by  
165 many publications addressing learning in ICT and in the area of business studies.

166 Michael Cole is a well established Vygotsky scholar, and James Wertsch also has  
167 his roots in Vygotsky although in developing his dialogical theories he also draws  
168 on Bakhtin and other Russians. Yrjö Engeström is a well known modern Activity  
169 Theorist. Engeström draws on Leont'ev's work, who is one of Vygotsky's leading  
170 followers. However Engeström extends Leont'ev's theorization by adding a third  
171 interacting entity, the community, to the two components, the individual and the  
172 object, in Leont'ev's original scheme.

173 What this account shows is that although all these cited theorists of the partic-  
174 ipatory orientation have at some point drawn on Vygotsky, they have diverged in  
175 applying his ideas. Furthermore, other theorizations not cited by Wegerif have been  
176 drawn on by participatory theorists, such as Foucault and other post-structuralists  
177 (Henriques *et al.* 1984).

178 Wegerif characterises the participatory orientations as sharing a concern with in-  
179 teraction in social contexts. Learners 'move' from the periphery to the centre of a  
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182 <sup>3</sup>Identity is also a theme in social sociology which is another root source for this burgeoning area  
183 of research in mathematics education.  
184

185 community of practice, in the sense of graduating from novice status to full partici-  
186 pants. This from this perspective learning is in the relationship between people and a  
187 particular environment. From this perspective transfer of knowledge is problematic  
188 because on the whole knowledge is tacit and socially embedded rather than explicit  
189 and moveable in terms of semiotic/textual representations. Education is about full  
190 participation in communities of practice and utilization of its resources, and the aims  
191 of the educator are working to establish communities of practice in which conversa-  
192 tion and participation can occur.

193 The trouble with this account is that it does not distinguish between learning  
194 mathematics in school or university, learning to process claims in an insurance com-  
195 pany (one of Wenger's examples), learning the 12-step programme in Alcoholics  
196 Anonymous (one of Lave and Wenger's examples), or learning to be a garbage col-  
197 lector on a specific route in North London (something I did as a youth). While there  
198 undoubtedly is learning taking place if you are a member of any of the last three  
199 communities or workplaces there are also major differences with learning mathe-  
200 matics. For virtually all students of mathematics immersion in mathematical prac-  
201 tices from the age of 7 or so until the break points at 16 years (high school gradua-  
202 tion), 18 years (end of pre-university specialist studies in mathematics) or even 21  
203 years (first degree in mathematics) does not constitute an apprenticeship in math-  
204 ematical research. Rather it constitutes a training in certain forms of thinking that  
205 will be applied across the full range of studies and occupations. So we need to dis-  
206 tinguish sharply between social practices that are a productive end in themselves  
207 and those that are simply a means to some other end, possibly undetermined during  
208 this preparatory activity. This is what education consists of.

209 Although this critique applies to the works of Lave and Wenger cited, the same  
210 cannot be said to some of the applicers of their theoretical perspective within the  
211 research community. Likewise researchers in mathematics education have drawn on  
212 Cole, Wertsch, and Engeström's work, as well directly on Vygotsky in accounts of  
213 social constructivism (e.g., Ernest 1994, 1998).

214 Undoubtedly the participatory perspective as put forward by Wegerif does offer  
215 something missing from traditional accounts of the teaching and learning of math-  
216 ematics as simply the passing on knowledge via representations. For although not  
217 all of mathematical knowledge is tacit, embedded in social practice, some of it is.  
218 In all fields of study much of our professional judgement and professional practice  
219 is based on 'knowing how it is done' rather than explicit rules or procedures that  
220 can be applied thoughtfully or mechanically. Even in mathematics judgements as to  
221 the correctness of a published proof or a student's written solution to a problem are  
222 based on implicit professional 'know how' acquired from practice (Ernest 1999).  
223 Kuhn (1970) makes this point forcibly for all of the sciences. According to his ac-  
224 count, at the heart of a scientific paradigm are examples of accepted reasoning and  
225 problem solving. It is the skilful following and application of examples rather than  
226 the use of explicit rules that constitutes working in the paradigm.

227 In Ernest (1998) (drawing on Kitcher 1984) I suggest that mathematical knowl-  
228 edge has a number of components that go beyond those traditionally identified.  
229 There are of course the traditional accepted propositions and statements of math-  
230 ematics, as well as accepted reasonings and proofs. Together with the problems and

**Table 2** Mathematics knowledge components and their explicitness

Mathematics knowledge component	Explicitness of component
Accepted propositions & statements	Mainly explicit
Accepted reasonings & proofs	Mainly explicit
Problems and questions	Mainly explicit
Language and symbolism	Mainly tacit
Meta-mathematical views: proof & definition standards, scope & structure of mathematics	Mainly tacit
Methods, procedures, techniques, strategies	Mainly tacit
Aesthetics and values	Mainly tacit

questions mathematics these make up the mainly explicit knowledge of mathematics. But I also argue that to know mathematics involves knowledge of its language and symbolism, knowledge of meta-mathematical views including proof and definition standards, and the scope and structure of mathematics. Such knowledge is mainly tacit or craft knowledge, embedded in practice. In addition, knowledge is also needed of the methods, procedures, techniques and strategies of mathematics as well as the aesthetics and values that underpin judgements in mathematics. All of these are mainly tacit, acquired from working in the practice of mathematics. These knowledge components and their status as explicit or tacit knowledge are listed in Table 2.

Thus some formulations of what Wegerif terms the participatory orientation do support a valuable account of the mathematical knowledge needed for mathematical practices, both by research mathematicians and by the learners of mathematics. But the participatory orientation is a broad church encompassing differing and sometimes, if not conflicting theories of learning, uneasy bedfellows to put under the same blanket. This is not to criticize Wegerif's (2002) use of broad brush strokes to distinguish the cluster of participatory orientated perspectives from the other three learning orientations. It is simply to say that the loosely clustered together perspectives under such headings are far from identical, and of course the same criticism can be directed at the four theory clusters in Ernest (2006).

All of the learning theories distinguished by Ernest (2006) and Wegerif (2002) downplay what I now take to be a vital element, as I mentioned earlier. This is ethics and values. So why are ethics and values so central to learning theories in mathematics education? My claim is that ethics enters into mathematics education research in four ways. First of all, there is a vital need to be ethical in our research. As responsible and ethical professionals, it is incumbent on us at the very least to ensure that our research is based on the informed consent of any human participants, does not cause them any harm or detriment, and that we respect the confidentiality and non-identifiability of all individuals or institutions. Any research that does not fully conform to ethical standards is not only ethically flawed, but its claims to add to the sum of knowledge must be viewed as suspect. Unlike stolen money which is just as good in the shops as honest money, unethically derived knowledge is epistemologically as well as ethically tainted.

277 Second, as educational researchers we are participating in the great, age-old hu-  
 278 man conversation, which sustains and extends our common knowledge heritage. By  
 279 sharing our thoughts, our findings both informally and formally, and through our  
 280 publications, we are part of the public conversation from which others benefit. This  
 281 great conversation, as Michael Oakeshott called it, is not a means to an end, but an  
 282 end in itself, and the conversation is inescapably moral and ethical. To participate  
 283 you must value the contributions of others. You must listen with respect and hu-  
 284 mility, and when you have developed a voice, you contribute to the conversation,  
 285 knowing it is much greater than you. The tacit values implied by participation are:  
 286 valuing and respecting the voices of others, past and present; valuing the young  
 287 who will get the chance to participate; not taking too seriously the trappings of  
 288 power, earthly prizes, ego gratification, these will all be gone and forgotten as the  
 289 great conversation rolls on; striving for excellence and high standards in oneself and  
 290 others—both to be worthy of the great conversation, and to protect it; recognising  
 291 that all human beings are part of this transcendent shared enterprise, and that all  
 292 members of the human family deserve concern and respect. Mathematics education  
 293 is one of the strands in the great conversation and we in its research community can  
 294 be proud that our efforts and those of our predecessors have created and swelled one  
 295 of the strands of this great shared enterprise.

296 Third, it is a self-evident truth that as human beings we are irreducibly social  
 297 creatures. Humans as a species are essentially interdependent. We emerge into the  
 298 world after our initial biological development within our mother’s bodies. We must  
 299 experience love and care from others in our early years to become fully functioning  
 300 human beings. We must acquire language<sup>4</sup> and acceptable behaviour with others to  
 301 participate in social life and practices. Without such skills we cannot survive and  
 302 further the human race. Our species depends for its very survival on our ethical  
 303 and cooperative behaviour with regard to our fellow humans.<sup>5</sup> In its highest form  
 304 this dependency is expressed as the principle of reciprocity, embodied in all ethical  
 305 belief systems and world religions as the Golden Rule: ‘Do unto others as you would  
 306 have them do unto you’ (Wikipedia 2009). One source for this is the awareness that  
 307 we are all the same but different (to paraphrase the title of Quadling’s 1969 book  
 308 on equivalence relations) and but for luck and contingency you and I as individuals  
 309 could be in each other’s situation.

310 Fourth and last, but far from least, prior to all such reflections, according to Lev-  
 311 inas, we owe a debt to others that precedes and goes beyond reasons, decisions, and  
 312 our thought processes. It even precedes any attempt to understand others. Levinas  
 313 maintains that our subjectivity is formed in and through our subjectedness to the  
 314 other, arguing that subjectivity is primordially ethical and not theoretical. That is to  
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 317 <sup>4</sup>By language I include all complex systems of human communication such as signing for the  
 318 hearing impaired.

319 <sup>5</sup>I am not so idealistic or unrealistic so as to ignore the recurring presence of competition and  
 320 contestation in human affairs at all levels. However, my claim is that human cooperation, mutual  
 321 help and care must exceed competition, contestation and antagonism or else as a species we would  
 322 have perished in the past or will perish in the future.

say, our responsibility for the other is not a derivative feature of our subjectivity; instead, this obligation provides the foundation for our subjective being-in-the-world by giving it a meaningful direction and orientation (Levinas 1981). This leads to Levinas' thesis of 'ethics as first philosophy', meaning that the traditional philosophical pursuit of knowledge is but a secondary feature of a more basic ethical duty to the other (Levinas 1969).

Thus one can say that as social creatures our very nature presupposes the ethics of interpersonal encounters, even before they occur, and even before we form or reflect on our practices, let alone our philosophies. This is why Levinas asserts that ethics is the 'first philosophy' presupposed by any area of activity, experience or knowledge, including mathematics education. If we accept his reasoning, then ethics is also the 'first philosophy' of mathematics education. It precedes any theorizing or philosophizing in our field, and this constitutes a hidden underpinning that precedes any discussion of, for example, theories of learning mathematics.

Unfortunately ethics as the 'first philosophy' of mathematics education tells us little specific about mathematics or the teaching and learning of mathematics, other than to respect and value our peers, students and indeed all peoples. But acknowledging the primordially social character of human beings weakens the claims of theories like behaviourism, simple constructivism, radical constructivism, enactivism and even humanistic psychology that are expressed in individualistic terms. If such theories do not take into account our irreducibly social character, there is a strong case that can be made against them. Thus, for example, radical constructivism's account of the learner as a cognitive alien making sense of a world of experience, and constructing other persons as regularities in that world, in effect denies the social and ethical foundation of human being (Ernest 1994).

Learning is not something that takes place in a social vacuum by any account, and socio-cultural theory and social constructivism prioritize the social environment as a primary element in the learning and of course the teaching of mathematics, thus becoming, on the basis of my argument, irreducibly ethical theories. However, it would be naïve to finish without acknowledging a possible rejoinder from proponents of individualistic learning theories. Namely that such theories because of their deliberately narrower focus do not dwell on the social or ethical, but as thinking tools for humans, like any other theories, they must be applied ethically. All human activities must take place under an ethical umbrella and the fact that ethics is implicated in social theories (albeit at one remove) does not give their supporters any free ethical 'brownie points'. For sociologists to claim that their area of study is more ethical than that of psychologists, not that they do so, would be arrogant, laughable and simply false.

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