



The *First* Sourcebook on Nordic Research in Mathematics Education

*Norway, Sweden, Iceland, Denmark,
and contributions from Finland*



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and contributions from Finland*

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*To my children Sarah, Jacob and Miriam & the North—
always true, inspiring and guiding...*

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FOREWORD

Bharath Sriraman

THE UNIVERSITY OF MONTANA

It is difficult to believe that a task that was once perceived as impossible by many skeptics has indeed been accomplished. The idea of a single comprehensive Sourcebook to showcase the mathematics education research done in the Nordic countries came to me a few years ago, and upon broaching this suggestion to those in the hierarchical orthodoxy that characterizes any field in various parts of the world during my travels in the last 7 years, I was told that it was too ambitious a project to orchestrate. Yet this book has come to Be!

My connection to the far North first began as a young sailor and continued as an undergraduate student in Alaska, a traveler in Yakutsk, Russia and the Canadian North, and has carried on in my professional life with numerous colleagues in Iceland, Norway, Denmark and Sweden, who have been like a wonderful extended family. Working professionally in North America for most of my career led me to the realization that a considerable amount of valuable research from “elsewhere” was often overlooked by researchers in the field taking a rather myopic view of the literature. Taking inspiration from Marcia Ascher’s influential book *Mathematics from Elsewhere* and from Claudia Zaslavsky’s classic *Africa Counts*, which are books that changed mainstream mathematicians’ perceptions of the heritage of mathematics, I thought it was of value to the mathematics education community *outside the Nordic world*, to pay attention to the considerable efforts of our colleagues in Norway, Sweden, Iceland, Denmark and Finland, to advance our field. Another reason for putting together a book such as this was to create a visible platform for researchers within the Nordic world to have an international audience become familiar with their work, since a large portion of the extant Nordic research, with some notable exceptions, has appeared within reports and journals that are not accessed by those “elsewhere.”

The book is organized in 5 sections, namely Norway, Sweden, Iceland, Denmark and Finland. Each of

the sections was compiled and edited by a researcher situated within the local national context. For the Norwegian and Swedish sections, I initially played a coordinating/organizational role after which I handed over the reins to the two section editors Simon Goodchild and Christer Bergsten respectively. The Norwegian section, compiled by Simon Goodchild, is the largest section of the book. Simon provided extensive feedback to the numerous authors in Norway, which occurred in numerous iterations to raise the quality of the presentation in the chapters. I received several e-mails from Norwegian authors who indicated that Simon’s guidance, skill and patience be recognized. The 17 chapters of the Norwegian section are the result of his dedicated effort for which they are very thankful. Christer Bergsten was entrusted with the Swedish section of the book that again required juggling a large cast of authors with varied backgrounds, and in his inimitable style Christer brought his section consisting of 13 chapters to completion with diligence and care. I thank both Simon and Christer for the two larger sections of this book.

The Icelandic section was put together by Gudbjorg Palsdottir and myself with advice from Gudny Gunnarsdottir. Mathematics education research in Iceland is in its infancy in comparison to the other Nordic countries, however Iceland’s infancy confers the advantage of learning from mistakes made elsewhere and being able to glimpse at developments on either side of the Atlantic [*alas their banking system did not do so!*]. Indeed, Iceland is located on the mid-Atlantic rift and as a result is geographically located both on the continents of North America and Europe. The 9 chapters in the Icelandic section reveal efforts aimed at mathematics teacher, researcher and curricular development. I feel privileged to have been privy to this for the last many years and thank all the authors for their willingness to write about their efforts in mathematics education. The Danish

section of the book was organized by Claus Michelsen and myself, but with the growing pressures of time and other factors, Claus handed over the section to Bettina Dahl, who then went on to produce a very impressive section of the book consisting of 10 chapters. As I write this introduction to the book in Tromsø, Norway—a conference to celebrate the work of Ole Skovsmose on his retirement took place in Aalborg, Denmark. Ole’s work on critical mathematics education is known outside the Nordic world and has inspired many researchers around the world including myself to carry on his bold vision for mathematics education. Finally, the Finnish section of the book contains four contributions compiled by Lenni Haapasalo, who is to be thanked for his effort.

This book taken as a whole serves as a valuable addition and complement to the present mathematics education library of Information Age Publishing which also published the *Second Handbook of Research on Mathematics Teaching and Learning* in 2007. I wish to thank the large cast of authors of this book for their patience and willingness to stick with this ambitious project and see it to its completion. In particular I would like to thank the following colleagues in the Nordic world for their camaraderie and support, namely Gudbjorg Palsdottir, Gudny Gun-

narsdottir, Olof Steinthorsdottir in Iceland; Simon Goodchild, Barbro Grevholm, Anne-Birgitte Fyhn, Raymond Bjuland, Reidar Mosvold in Norway; Claus Michelsen, Bettina Dahl in Denmark; and Christer Bergsten in Sweden, among others. Last but not least the support and faith of George Johnson, the founder of Information Age Publishing, in this book project is much appreciated. The Montana Mathematics Enthusiast Monograph Series in Mathematics Education is pleased to present this massive compilation of 53 chapters focused on Nordic mathematics education research to the reader. I hope this book is of use to many generations of mathematics education researchers inside and outside the Nordic world.

Tromsø, Norway
January 17, 2010

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