The Mathematics of Estimation: Possibilities for Interdisciplinary Pedagogy and Social Consciousness

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ABSTRACT: The purpose of this article is to report on the importance of providing pre-service and in-service teachers with experience and specific training in critical thinking skills. The essential concepts in elementary mathematics curricula can be augmented to include and cultivate critical thinking skills that have tremendous ramifications for future leaders and for those who move on to more technical training. A sample problem, along with pre-service teacher responses, is used here to show the necessity and importance of this kind of training as the responses show clear evidence of a certain naivete on the part of these college level students. The responses do show evidence of budding social conscience in the students, but the level of expertise in critical thinking is not at all sophisticated. We discuss and explore the implications of our approach.

KEYWORDS: Ecological problems, estimations, permi estimates, interdisciplinarity, interdisciplinary pedagogy, mathematics education, materialism, proportional reasoning, pre-service teachers, resource consumption, social consciousness, sustainable development.

Introduction

The 21st century student lives in an age where societies are inundated with information from various multimedia. It is not uncommon to find elementary school students who are versatile in using cellular phones and savvy about accessing the world wide web. Given the over abundance of information sources, it is necessary for students to be able to think critically and examine the validity of the information they are receiving. Mathematics has an important place to play in the modern world particularly when it comes to making sense of the barrage of

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quantitative information which has become of part of everyday life. The first question we ask is what types of elementary mathematical ideas and/or thinking skills are most useful in the day-to-day world? With this question we seek to connect the standard curriculum to practical uses of mathematics and critical thinking skills. We contend that (a) reasoning in ratios, (b) estimation, and (c) problem solving are arguably the three most important types of mathematical thinking skills from a K-8 teacher perspective. Numerous pedagogical papers call for the increased use of problems situated in the real world to promote higher order mathematical processes such as reasoning, abstraction, and generalization (e.g., Doerr, 2006; Doerr & Lesh, 2003; Doerr & English, 2003; Freudenthal,1991 ; Sriraman & Lesh, 2006) .

Hans Freudenthal (1991), the mathematician turned mathematics educator was instrumental in initiating a view of mathematics education with a particular concern for everyday context or the larger reality within which students' lives are situated. In the Realistic Mathematics Education (RME) movement two essential criteria for the teaching and learning of mathematics are set out. First, mathematics must be close to children and be relevant to every day life situations. However, the word *realistic*, refers not just to the connection with the real-world, but also to problem situations which are real in students' minds. For the problems to be presented to the students this means that the context can be a real-world but this is not always necessary. De Lange (1987, 1992) stated that problem situations can also be seen as applications or modeling. Second, the idea of mathematics as a human activity is stressed. Problem solving and problem posing have been both identified and justified as areas of importance by various curricular documents (e.g., National Commission on Mathematics and Science Teaching – NCMST, 2000; National Council of Teachers of Mathematics – NCTM, 2000; National Research Council – NRC, 1998) as well as in recent mathematics education works (e.g., Gravemeijer, & Doorman; Lesh & Doerr, 2003; Sriraman & Lesh, 2006). Yet there is inevitably a gap between the curriculum (and its implied intentions) that teachers use and their actual classroom practices which ultimately affects both student learning and the value derived by students about mathematics from problem solving.

The following philosophical considerations may help in bridging that gap. The Greek philosopher, Socrates, once said that our ultimate purpose is to will *Good* for humankind. If one accepts this premise and connects this to the purpose of contemporary education (and not only the discipline of mathematics), then it becomes clear that two broad goals of education must be:

(a) to produce citizenry who are capable of thinking critically and are willing to engage in such thought; and

(b) o develop an awareness for the value of making reasoned choices that seek to will Good for humanity.

Critical thinking is often not associated with the teaching and learning of mathematics, however the two disciplines share many common traits. Historically, training in critical thinking makes explicit use of formal logic in order to draw inferences and/or make comparisons. Mathematics can be presented as being structured and rigid in the same way, but this need not be so. In *Apology* Plato suggested that one should not blindly accept a persuasive argument without being aware of the reasons why the argument is persuasive (1999). In other words, a critical thinker must be able to examine the validity of the logic used in an otherwise eloquent and persuasive argument, as well as to verify the facts and assumptions that are involved. Likewise, students of mathematics can be taught to question the didactic claims of their teachers and can be taught to validate mathematical propositions based on their own emerging skills and frames of reference. For the ancient Greeks, critical thinking not only involved an examination of the eloquent words and actions of other people but also an examination of one's own thoughts and capabilities. The traditional constructs of critical thinking have been criticized as being "a narrow way of thinking, excessively centered on reasoning and argumentation" (Smith, 2001, p. 349) which do not take into account imagination or intuition, and do not nurture the creative (generative) side of thinking (Walters, 1994). If we believe well-stated arguments based on the position of Smith, Walters, or the authority of the teacher, then we can easily be misled by charismatic voices and thus fail to question and think critically for ourselves. Another criticism about the traditional view of critical thinking is that the excessive focus on formal logic, rhetorical ploys, fallacies, and argument construction encourages students to view critical thinking as merely an arduous mental exercise without any wide-ranging applicability (Adler, 1991; Baron, 1988; McPeck, 1984). Students may lack the confidence to challenge propaganda or advertising because they might feel the rigorous tools required to think critically are beyond their abilities. Likewise when mathematics is taught as formal algorithms, with learning restricted to successful computation without any requirement to apply this mathematics to the real world, then it weakens the growth of knowledge for students (Sriraman & Adrian, 2004). Students must learn not only to perform in the context of their own world, but to explain what they are doing and why it is important.

Given these criticisms of the traditional definitions of critical thinking, we adopt a modified view of critical thinking that is compatible with the expectations we generally hold for beginning students. We define critical thinking as “reasonable reflective thinking that is focused on deciding what to believe or do” (Ennis, 1991, p. 6) with the added requirement that it be connected to real life. This pragmatic view enables students at very early stages to understand the cultural and instructional influences that ought to influence accepted thought (Bacon, 1620/1902). The rationale for choosing this definition is that it requires that critical thinking skills apply to real world problems, brings to the forefront the issue of bias in critical thinking (Paul, 1990), and makes use of appropriate questioning to stimulate students’ reflections on problems (Simpson, 1996).

Arguably, there are no curricular traditions or widely adopted educational proficiency standards that give educators a recipe on how to inculcate this skill set in their students. However, university educators, especially those engaged in the training of future teachers, are in the enviable position of influencing the overarching ideals these future teachers carry to their classrooms and students. In particular, the most elementary ideas from K-8 mathematics can be broadened to include a useful and constructive critical thinking perspective and thus illuminate the problems confronting people in all societies in our shrinking globe.

The “Math Wars” (in the United States) and similar differences of curricular opinion elsewhere show that several schools of thought exist about the place of mathematics in the curriculum. On the one hand traditionalists emphasize the necessity of teaching and learning the basic skills of mathematics, that is, rudimentary arithmetic and algorithms, in order to develop the mathematical tools necessary to investigate open-ended problems. That is: why set up students for failure by having them tackle or pose real world problems for which they do not have the necessary mathematical tools? The non-traditionalists (reform educators) on the other hand de-emphasize excessive focus on basic skills (to the exclusion of other lessons) and call for a greater emphasis on the teaching and learning of the concepts underlying procedures and algorithms. This may take more teacher time initially, but will result, they argue, in better retention of mathematics content because it becomes relevant and useful in the day-to-day lives of students. The rejoinder to the caution of setting students up for failure is that in problem-solving situations there are many opportunities for the development of critical thinking skills and the teaching of contemporary mathematical tools at an appropriate rate of development. The traditionalists typically have numerous rejoinders to this counter-argument. They assess their students and find them to be deficient in the rudimentary skills because the students have not learned from the worksheet/rote lessons they have had to endure. If students cannot multiply fractions, for example, they say, how can students be expected to question theorems or apply the technological tools to run regressions or master other applied mathematical concepts? Yet without an understanding of the underlying concepts of mathematics at each stage, how can students reach out and apply what they learn to the real world? So the "Math Wars" continue unabated. This debate has proven to be intractable as each side is entrenched in what they believe is good for the students and yet each side emphasizes elements which constitute a Hegelian dialectic so to speak. The progress of teaching well in a historical perspective is slow at best. This dialectic is evident in Schoenfeld's (2004) historical perspective of the "Math Wars" in which he distilled the underlying premises and issues under question. "Is mathematics for the elite or for the masses? Are there tensions between 'excellence' and 'equity'? Should mathematics be seen as a democratizing force or as a vehicle for maintaining the status quo?" (p. 253). All these questions have been the subject of recent mathematics education research (e.g., Applebaum, 1995; D'Ambrosio, 1990, 1994a, 1994b, 1998, 1999; Gutstein, 2006), which calls for mathematics to ultimately promote a sense of social justice and empowerment. How do we get there?

Perhaps the time is ripe to create a completely different perspective, one that does not attempt a postmodern resolve of the two entrenched sides. Postmodernism would disassemble all mathematics teaching and return it to the basics, rather than build on the evolving successes that have already been achieved. What we need instead is an approach that creates a paradigmatic and humanistic shift about the purpose of mathematics in today's global economies and societies. This involves emphasizing the three key, identified skill sets: (a) reasoning in ratios, (b) estimation, and (c) problem solving in the context of the culture and communities that surround the students. The National Council of Teachers of Mathematics (NCTM – 2000) asserts:

> Estimation activities encourage students to make connections among the mathematics concepts they are learning and the skills they are developing … the class discussions and the decisions the

> teacher makes contribute to students' opportunities to connect their understandings of number, measurement, geometry, and data in order to make estimates. (E-chapter 4, section 4.6)

It further states that purposeful activities along with skillful questioning e.g. improvements in the use of appropriate kinds of discourse (Applebaum, 1995; Knott, Sriraman & Jacob, 2006) promote the understanding of relationships among mathematical ideas. In fact, this recommendation can be pushed a step further and estimation activities can be used to promote both the usefulness and development of mathematical concepts (and procedures) and cultivate critical thinking. This is particularly important at the college level for pre-service teachers if we wish to create informed classroom teachers who are aware of larger issues that can be tackled via mathematics.

From Critical Thinking to Mathematical Literacy to Critical Pedagogy

The role of critical thinking has been re-iterated in the modern world with recent concerns for curricula and pedagogy which emphasize mathematical literacy. The Organization for Economic Co-Operation and Development (OECD, 2004) defines mathematical literacy as an individual's capacity to identify and understand the role that mathematics plays in the world. Further literacy involves making well-founded judgments (thinking critically) and using and engaging with mathematics in ways that meet the needs of each individual's life as a constructive, concerned and reflective citizen. However, in spite of the good intentions of the OECD, the push for mathematical literacy does not address pedagogical practices (How do we get there?) and goals which promote greater social awareness or a social consciousness appropriate for initiating major shifts in thinking. Freire (1998) suggested that pedagogical practices should support education for liberation and emphasized problem-posing pedagogies that strive "for the emergence of consciousness and critical intervention in reality" (p.62).

A Pedagogical Example

Can such an idealistic vision be realized? The need for this kind of paradigm shift is demonstrated by the results of the following contextual estimation activity. Pre-service elementary teachers (henceforth called students) were presented with such an assignment. These are students in a mathematics content course for elementary school teachers, and they were presented with the following statement after having been shown the importance and power of proportional reasoning, estimation and problem solving as tools for sense making.

> Total wastes in the United States, excluding wastewater amounts to approximately 50 trillion pounds a year.[1]

The sheer magnitude (and absurdity) of the number resulted in students suggesting the need for a different perspective of the quantity 50 trillion pounds. Assuming the population of the United States was 300 million, using ratio and proportion concepts we calculated

$$\frac{50{,}000{,}000{,}000{,}000\,lbs}{365\,days} = \frac{50{,}000{,}000{,}000{,}000}{365\,days \times 300{,}000{,}000\,persons} \approx 465\,lbs/person/day$$

In other words the 50 trillion pounds of waste per year translated to approximately 456 pounds of waste per person per day! This scaled down but still staggering figure in a classroom situated in an environmentally conscious community in Missoula, Montana raised more than a few eyebrows. It naturally led students to formulate questions such as: (a) is this figure believable? (b) if the figure is indeed true, how can it be made sense of? (c) where does all this waste go? (d) at what rate do landfills get full? (e) how does this figure compare to the waste produced by Missoula? (f) how can we estimate the waste produced by Missoula, knowing that we are an environmentally conscious community? (g) what portion of the United States will be landfills if we continue to produce waste at this phenomenal rate?

These questions are not only mathematical but also reveal the critical thinking of the students and their larger concern for the well being (Good) of the citizenry and the environment. The most important question posed was whether this figure was believable. A *student-initiated assignment* attempted to answer these questions. In the remainder of this paper, examples from student work and reflections are presented which reveal the power of basic estimation techniques situated in this context. Student work, which combines mathematics and critical thinking, reveals the will to do Good, and also their naiveté, armed with this new perspective created by simple mathematics. It also indicates their ability to think and argue both sides of an issue via mathematics and come to a reasonable conclusion about the societal and environmental impact of the waste problem. Students were given two weeks to make sense of the figure presented in class and to construct a written report.

Rebecca's Analysis:
A City as a Particular Case With Data From the Web

In order for me to decide whether the figure we calculated in class was believable, I looked up the statistics for the city of Missoula available on the web. I found that approximately 240,000 tons of trash per year is produced in Missoula. There are also approximately 99,000 people in Missoula including the transient population. So,

$$\frac{240{,}000\,tons}{1\,year} = \frac{240{,}000\,tons \times 2240\frac{lbs}{1\,ton}}{365\,days} \approx 1{,}470{,}000\,lbs/day$$

So, the figure per person is

$$\frac{1{,}470{,}000\,lbs/day}{99{,}000\,persons} \approx \frac{15\,lbs/day}{1\,person}$$

I think my answer is reasonable especially compared to the number that we calculated in class of 456 lbs/day for the country. I rounded several figures, such as using 99,000 instead of 98,813 for the population. My answer only took into account Missoula and in spite of rechecking, I could not come up with a bigger number. Some other sources report the population as 70,000. In this case the figure is 21 lbs/person/day, which is again way lower than 456 lbs per day. The only way I can imagine this statistic is accurate is we took into account trash production of businesses and manufacturing companies. Still I doubt that 456 is really true.

Shaylene's Analysis: A Person as a Particular Case

A person eats three meals a day which is approximately 4 lbs of trash in leftovers, wrappers, packaging, and so on. A person also has two small snacks which produces another 1 lb of trash. A person also disposes papers and miscellaneous trash at work/school/home which is another 3 lbs of trash. The use of public facilities (restrooms, eating out, coffee, etc.) causing these facilities to produce trash for the person is about 4 lbs. Add another 2 lbs for trash produced at random for unmentioned reasons (say going to a party, cleaning house, and so on). This adds to 14 lbs of trash per day per person living in Missoula. This is lower than that of 456 lbs of trash that we discussed in class. It is hard to believe that the figure from class is correct because if it were correct, it would mean an individual is throwing things out at all times of the day.

Adriana's Analysis:
Summing the Parts With Reasonable Assumptions

I cannot believe that the amount of waste broken down averages out to 456 lbs per person per one day. That really makes you think about how much we really waste and that we need to change our consumer lifestyle. In order to find the amount of waste generated by a person in Missoula each day, I did the following calculation:

- Assume each person contributes at least 1 lb a day.
- There are 70,000 people in Missoula. So that adds to 70,000 lbs right there. . . . . [A]

Next to calculate the amount of food wasted by restaurants each day, assume that one plate of food has approximately 30 bites of food. People typically do not eat all of their food. Let us assume that about $1/6^{th}$ of each plate is wasted, so that is 5 bites. Assume that an average group that goes out to dinner is 4 people. Dinnertime is approximately 4-9 p.m. (5 hours). A meal takes about an hour to finish, so every hour a new group can sit at a table. In a 5-hour shift 5 groups are served per table. An average restaurant has 30 tables in it. There are 200 restaurants in Missoula (as seen in the Yellow pages).

So, the number of plates of food wasted at dinnertime in one night in Missoula is:

- 5 bites wasted × 4 people = 20 bites per group
- 20 bites per group × 5 groups per table = 100 bites per table
- 100 bites per table × 30 tables = 3000 bites per restaurant
- 3000 bites per restaurant × 200 restaurants = 600,000 bites in Missoula
- There are 600,000 bites wasted and 30 bites a plate, so

$$\frac{600{,}000}{30} = 20{,}000\,plates$$

- So, in any one night 20,000 plates of food are wasted.
- Assuming each plate of food is 1 lb, this gives 20,000 lbs per night.

- There is also extra waste besides food. Suppose each restaurant produces 50 lbs of trash, we get 200 × 50 = 10000 lbs of trash.
- So, the total trash for restaurants is 20,000+ 10,000 = 30000 lbs . . . . [B]

Healthcare facilities generate a lot of trash because they throw everything out. We have 2 hospitals, 5 clinics and about 10 rest homes.

- Assume 1 dumpster of garbage per floor in all these facilities. There are a total of about 22 floors, so 22 total dumpsters.
- Dumpster dimensions are 4 ft (depth) × 4 ft (width) × 8 ft (length) = 128 cubic feet.
- Each cubic foot of trash equals 10 lbs, so each dumpster holds 1280 lbs of trash.
- So total wastes produced by hospitals = 22 dumpsters × 1280 lbs/dumpster = 28,160 lbs of trash. . . . . [C]

We also need to factor in wastes produced by schools, and businesses.

- There are 5 high schools, 4 junior high schools and 10 elementary schools.
- Assume each school generates 1 dumpster of trash each day (being conservative).
- This gives 19 dumpsters × 1280 pounds/dumpster = 24,320 lbs from schools. . . . . [D]

There are 299 yellow pages full of businesses in the Yellow pages of the phone book. By counting the number of businesses per page for three random pages, I found there were 62 businesses, and 40% were from Missoula. About 5 of the pages were for restaurants, which we calculated already.

- So [299 pages – 5 pages (for restaurants)] × 62 businesses/page = 18228 businesses.
- 40% of 18228 businesses =7291 Missoula businesses.

Some of these businesses are large corporations, some are privately owned small businesses. Assume these average out and produce a dumpster a day. That produces:

- 7291 dumpsters × 1280 lbs/dumpster = 9,332,480 lbs of trash! . . . . [E]

Adding all five categories [A] through [E], we get:

- 70,000 + 30,000 + 28,160 + 24,320 + 9,332,480 = 9,484,960 lbs.

This averages to $$\frac{9484960 lbs}{70000 persons} \approx 1.36 lbs/person/day$$

This number seems logical although not as close to the 456 lbs per day. There are size differences in cities which have large populations living together in high rises and so on. It is still unbelievable to think each of us is responsible for 136 lbs of trash each day.

Discussion

The three solutions are proto-typical of the 19 different solutions received from the students. Rebecca's strategy was employed by seven other students, who mined the world wide web for population and trash data for the city of Missoula and computed their answer via a simplistic proportion. Shaylene's strategy was similar to strategies used by five other students. These students used themselves as a particular case, and conducted experiments like calculating the amount of trash produced by their families or dorm room-mates to arrive at a personal figure for trash production per day. This figure was then compared to the figure of 456 lbs. Adriana's strategy was employed by five other students. The approach of these students was to make reasonable assumptions for trash produced by various sources in the city to arrive at a figure for each person. This was the most sophisticated strategy in the classroom.

In general, solutions had estimates ranging from 4 lbs/person/day (based on the analysis of a trash-can in a student's room and keeping track of other waste produced by a student each day) to 136 lbs/person/day. The diversity in student strategies and their solutions set the stage for a discussion of reasons why the individual calculations varied so drastically and why none of the solutions reached close to 456lbs/person/day. One of the students suggested computing the average of all the solutions which was approximately 60lbs/person/day.

The ensuing classroom discussions were focused on critically examining (a) the assumptions made by students in arriving at a solution, (b) the reliability of sources of information, and (c) whether the figure of 456 lbs/person/day was plausible and if so why? Further problems which were suggested for exploration were:

- What is the area and volume of a landfill where trash/waste is disposed?
- At what rate do landfills get full?

- What proportion of the area of the U.S would be landfills if we produced trash at this rate for the next 100 years.
- Are there differences in trash production between the United States and other countries (countries suggested were Australia Canada, China, France, Switzerland, India). How can differences be explained?

Student Reflections

In this section, we present contrasting student reflections during the course of this pedagogical exercise.

> *Our class estimates were off for several reasons. First we need to bear in mind that we did not factor in daily industrial waste, which can only be accurately obtained from sources like the EPA or other national environmental groups. Second the figures need to be adjusted according to the proportion which is recycled. It is reasonable that our figures are less than the national average. For instance if we compared the waste per day produced in Missoula to that of New York City based on the figures available from Fresh Kills Landfill in Staten Island, we can see that large cities which are densely populated produce more trash. ... [t]hey have more hospitals and other public services necessary to serve the population. Incidentally, the Fresh Kills landfill is now closed and I found accurate figures about its size and capacity. This landfill was a 2200 acre site with 6 landfill mounds and lasted about 55 years with trash being primarily household waste. In contrast the Missoula Landfill is much smaller in size, about 56 acres, and its depth is only 85 feet. It opened in 1968 and is slated to close in 2019 ... again a life period of about 50 years. So landfills do not really get full very quickly as some predicted. If we actively recycled, the rate at which they get full could be further slowed. This requires we consume less and recycle more.*

> *Technology will find a way to solve the trash problem. In the worst case scenario, if we kept producing trash at the rate of 50 trillion pounds a year, and assuming landfills can be made about a mile deep, a mile long and a mile wide. ... [t]hat gives about* 1.48×10^{11} *cubic feet of volume. I think that 10 pounds can be compressed roughly into 1 cubic feet. So we have 50 trillion pounds, which will*

take about 5 trillion cubic feet or 5×10^{12} *cubic feet, which is nearly the capacity of this landfill. Now there are 3,717,796 square miles of area in the U.S, and even assuming that 5 square miles of area get used up by landfills each year, in about 100,000 years, we would only have used up only*

$$\frac{5 \times 100{,}000}{3{,}717{,}796} \approx 13\%$$

of the land. If the U.S. makes it that long there will be technology to make trash vaporize. It is really not such a big deal as people are making it seem.

Our country is one of the largest, if not the largest creator of garbage in the world. If something is new and the neighbors have it, then we need to buy it and dispose of the old. We buy and throw more than many countries have or will. Yet we complain about overpopulation in other parts of the world not knowing that they produce a tiny fraction of what we do per year. ... .[B]ut in defense of the country, there is a greater amount of recycling of our garbage in the past ten years and if we keep this up and people compost and recycle the amount of garbage they produce, our landfills would go down even further.

I never imagined learning about garbage in a Math class! But I am glad I did. It really makes me want to cutback on all of the garbage I throw away. I hope there are others in the class that think the same. Gosh we need to stop being so wasteful ... it is so embarrassing to look at ourselves.

It is actually deceiving to believe that each person actually contributes that much garbage daily because there are so many other ways garbage is produced that has nothing to do with an individual's consumption and wastefulness. It may be that environmentalists are trying to create a panic by only showing one side of the information.

Avenues for Critical Thinking and Concluding Points

Given the backdrop of such an activity, there are numerous possibilities for follow up activities involving critical thinking. Several such options are discussed based on the directions of in-class discussions. The first avenue could be to discuss the reliability of the sources of information, particularly the reliability of the given figure of 50 trillion pounds to begin with. Only two of the 19 students questioned the reliability of this figure. Another fruitful avenue of discussion that leads to critical thought was an exploration of biases in student approaches as a function of their family background (agrarian, rural, or urban); occupation of influential family members (parents, relatives, siblings); the media they were exposed to (traditional/mainstream versus alternative medias). One of the discussions focused on carefully examining the non-uniformity of assumptions made by students in their estimates. Consider the following quotes from students in their final reflective writing assignment on this problem which reveals awareness of biases and critical thinking:

> *Waste in America is just about as noticeable as the air we breathe. For most people here (in Missoula) the problem of waste does not factor into life and they do not have to deal with it on daily basis. The problem will be viewed differently depending on where one has lived. Having grown up on a ranch and having to actually deal with recycling waste is different from people here who just put away their trash each week, drive gas guzzling cars but support environmental charities. There is definitely a lot of hypocrisy out there.*

> *Having grown up in Missoula with parents that were hippies, I was constantly around people who were radical environmentalists. Sure we recycled as much as we could, had composts for the garden, but we also used the convenience of disposing trash like everyone else. You would have to like living in the woods independently if you wanted to free yourself from this problem but I don't think this is the point. The point is how can we live in cities and still maintain a sustainable life style without over impacting the environment.*

> *It seems to me that one thing the class completely overlooked in their over-zealousness was the fact that Land fills can be turned into parks or even housing developments after a certain time period. The garbage problem can certainly be managed if all the stakeholders within a community can reach a consensus on what percentage of wastes are recyclable and what needs to go into landfills. We can solve this problem locally.*

It is evident from the three solutions that much of the information used by the students in this problem came from personal estimates backed up by scant bits of actual data. Even then much of the data was based on their own personal experience. Although this is a good (yet naïve) place for the students to start, it is important that follow up activities use more reliable data from different sources. Many students seemed to have a collectively narrow and uninformed worldview and missed many aspects of garbage production in the "real world." For example, there was no mention of businesses (other than the restaurant business) which produce much of the garbage in Missoula.

The idea of working up to an estimate from personal individual use and habits, versus scaling down from the given figures shows a very inductive method of thinking, and emphasizes a basic difference in thinking, inductive versus deductive.

Implications

This paper is based on the premise that education should strive to promote interdisciplinary *connections* with other domains of knowledge relevant for today's world and include pedagogy that promotes social change. Connections is emphasized as one of the curricular goals in numerous documents. However, this is typically interpreted as connections among mathematical topics, that is, intra-mathematical connections as opposed to connections between mathematics and other disciplines. While the first interpretation is important, we prefer the latter, a broader interpretation of this term. The school curriculum in the United States is typically administered in discrete packages. The analogy of mice in a maze appropriately characterizes a day in the life of the high school student, with mutually exclusive class periods of math, science, literature, languages, social studies, and so on with the shibboleth being Advanced Placement exams. Yet reality does not function in this discrete manner. Critical thinking, problem-solving, and communication are real world skills that cut across the aforementioned disciplines (Sriraman, 2003). We also think that such activities need to be initiated at the elementary and middle school levels to cultivate ways of thinking. As students get exposure to more mathematics and science, they will begin to apply their knowledge of additional facts to check the credibility of their conclusions. The Conference Board of the Mathematical Sciences (CBMS) states "in many cases … teachers have been prepared to teach elementary school mathematics and lack the broader background needed to teach the more advanced mathematics of the middle grades" (2001, p. 25). We interpret this criticism as a call to increase interdisciplinary connections in the training of future elementary and middle school teachers. In this paper we have presented an innovative approach to pedagogy, one that fosters both mathematical and critical thinking of prospective elementary and middle school mathematics teachers.

Can Mathematical Problems Foster Critical Thinking? A Case for Fermi Problems

In science, particularly in physics Fermi problems are estimation problems used with the pedagogical purpose of clearly identifying starting conditions or assumptions and making educated guesses about various quantities or variables which arise within a problem with the added requirement that the end computation be feasible or computable by hand. The classical problem which states how many piano tuners are there in Chicago is attributed to the physicist Enrico Fermi after whom such problems are named. We argue that Fermi problems which are directly related to the daily environment are more meaningful and offer more pedagogical possibilities than purely intellectual exercises such as computing the number of piano tuners in a city or the number of grains of sand in a glass. Fermi problems involving estimates of fresh water consumption, gasoline consumption, wastage of food, amount of trash produced, and so on have the potential to lead to a growing awareness of ecological problems related to the environment we live in as well as provoke critical thought when checking the accuracy of computations with different governmental and corporate resources. There are several other possibilities for Fermi problems involving estimates of ecological parameters that have arisen due to the overuse and misuse of natural resources. We have used problems in which students make Fermi estimates of fresh water consumption and daily trash production, which inevitably lead to a growing awareness of resource usage and impact. This is brought about by critical thinking about the different estimates, their accuracy in comparison to various information sources and their impact on day-to-day life. Students begin thinking about solutions to problems brought about by the new found awareness of the threat to our planet because of over-population and overuse of natural resources. Other powerful problems which students' can be led to pose via such pedagogy are comparisons of resource usage in different countries. Our throwaway society and the idea of planned obsolescence impacts both the students thinking about the garbage problem and the amount of garbage produced by society in the United States. If we throw into the discussion the effect of advertising and the whole idea of consumerism and stir, the implications for interdisciplinary pedagogy become significant. Imagine if a math/science/sociology team collaborated on an exercise of this sort and made it the focus of a unit (week-long?) piece of the curriculum. Students would learn so much more about their world, how to live in it, deal with it, understand it, influence it, make it a better world. What a concept for education!

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NOTES

1. http://www.mindfully.org/Sustainability/Hawken-Resource-Waste-htm

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