CRITICAL NOTICE

On the Identit(ies) of Mathematics Education

BHARATH SRIRAMAN
The University of Montana

A critical notice of:

- Alexander, P.A. & Winne, P.H. (Eds.), *Handbook of Educational Psychology* (2nd ed.). Mahwah, NJ: Lawrence Erlbaum & Associates. (2006), 1055 pp. $89.95.
- Lester, F.K. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning*: *Part I – Foundations*. Reston, VA: National Council of Teachers of Mathematics & Information Age Publishing. (2007), 1324 pp. $350.00.
- Campbell, J.I.D. (Ed.), *Handbook of Mathematical Cognition,* New York, NY: Psychology Press, An Imprint of Taylor and Francis. (2004), 508 pp. $125.00.

ABSTRACT: This critical notice reviews sections of three recently published Handbooks of relevance to the community of educational psychology, cognitive science, and mathematics education researchers. In particular it examines (a) the overlap between the historic foundations of cognition, mathematics education, and educational psychology; (b) the shared problem of identity; (c) areas of consonance and dissonance, particularly the need to bring in socio-political issues; and (d) the complexity of understanding and modeling human cognition.

Prologue

"Handbook, Handbook on the wall,
Which of you is the handiest of all?"

The quote above is a play on the infamous words of the wicked step mother in the Grimm's fairytale *Schneewittchen (Snow White)*, "Mirror, mirror on the wall, who is the fairest of them all?" Even though the question is rhetorical, does anyone use the Handbooks at all, the answer that the mirror gives is objective, sometimes to the chagrin of the questioner. The question in the quote is posed to the three handbooks on the reviewer's wall and the answer (if one exists) is sought in this review. The aim of this *atypical* review is (a) to question and examine

Interchange, Vol. 40/1, 119–135, 2009.
DOI: 10.1007/s10780-008-9084-x

the identity of mathematics education as a field; (b) to question the role that Handbooks play in consolidating, filtering, and transmitting knowledge to the field; (c) to focus on the bigger questions confronting the field, namely coherence in theoretical frameworks, methodologies and implications for future research, policy and practice; and (d) to explore the possibility of Handbooks opening access to communities of scholars that have been unable to access academic journals due to high costs, other constraints, and affordances.

The field of mathematics education today is experiencing a radically fertile period as seen in the rapid increase and proliferation of journals, books, professional societies and conferences, monographs, and handbooks in addition to the increased use and relevance of theories, methodologies, and paradigms borrowed from fields other than psychology such as anthropology, activity theory, communication studies, complexity theory, linguistics, and sociology. Globalization and the Information Age have also played a significant role in both easing and increasing access to scholarly products that the field produces as well as decreasing the "half-life" of the cycle of manuscript submission, review, and production.

Yet scholars who have commented on the state of the art of the field of mathematics education have *both* praised it for its ever increasing heterogeneity and at the same time criticized it for lack of focus, the inadequacy of theoretical frameworks, or its ability to solve problems confronting the field (Lesh & Sriraman, 2005; Lester, 2005). Even though this may seem to be a paradoxical state of affairs, the three Handbooks which are purposefully chosen and reviewed here indicate that the view of our field depends on the scale that one chooses. Just like a fractal reveals self-similar patterns, increased complexity, attracting and repelling points when one increases the magnification factor or the scale that one chooses, the field of mathematics education reveals this self-similarity, complexity, and focal points of attraction and repulsion with various other fields when one moves from a general over-simplified view of the field to views afforded from changing the scale of one's perspective.

The *Handbook of Education Psychology* (2nd edition), henceforth HEPs, succeeds in giving the naive reader a general view of the field of mathematics education with its historical attraction to psychology. The *Second Handbook of Research on Mathematics Teaching and Learning*, henceforth SHRMTL, is the contemporary and more expanded version of the first Handbook which was released in 1992, and focuses on developments in our field over the last 15 years with emphasis in the areas of foundations, issues facing mathematics teachers and learners, teaching and learning outcomes, assessments, and international views. Finally, the *Handbook of Mathematical Cognition*, henceforth HoMC, reveals the complexity of research that address the biological, neurobiological, and neuropsychological bases of number concepts and processes, when one focuses in on specific sub-domains of educational psychology, namely the development of number concepts.

Curiously enough, the HEPs and SHRMTL overlap considerably in their foundational literature, issues of prescience, as well as in the list of authors and references to those currently involved in mathematics education. The *Handbook of Mathematical Cognition* (HoMC) is more specialized but in ways that bring out the complexity of specific questions raised in HEPs and SHRMTL on cognitive development with particular emphasis on specific mathematical content. *Caveat emptor* – The reader is warned that the style is discursive and biased towards issues and chapters from the three handbooks that I think are relevant and prescient for the field. I don't believe in simply regurgitating the table of contents of a book and calling it a book review.

Educational Psychology and Mathematics Education: Potential for a New Symbiosis?

Handbooks are by their very nature tomes of syntheses, containing erudite and comprehensive analyses of the state of research in a given field. They add a scholarly air to any personal library, albeit they may only get dusted off and referred to only occasionally. The *Handbook of Educational Psychology* (HEPs) is a 1000+ page tome which presents a coherent state of affairs on myriad domains of research within educational psychology in 41 chapters in eight sections written by over 80 scholars. The contents and the ideas in this book are very relevant for mathematics education, even though mathematics education per se is represented by only one of the 41 chapters within the book. This relevance is evident when one peruses the subject index and finds a large proportion of keywords found within the mathematics education literature. The symbiotic nature of the book with mathematics education also becomes evident when one reads through the author index and finds references in numerous chapters to well known papers within our field written by scholars like Cobb, Dienes, English, Fuson, Greeno, Greer, Hiebert, Lesh, Lester, Schoenfeld, Yackel, among others. Although the field of mathematics education has taken a social turn since the early 1990s with less emphasis on theoretical and methodological frameworks from psychology, I would venture to say that the book has something relevant for nearly everyone within mathematics education, including those interested in socio-cultural and political perspectives.

Part I of HEPs examines the foundations of the discipline of educational psychology with the first two chapters written by David Berliner and Robert Calfee, the editors of the first edition of HEPs. Berliner constructs a historical case study of the field of educational psychology beginning with Democritus' musings in 500 B.C.E on the nature of schooling and learning, the Roman conceptions of schooling during the time of Quintilian (35-100 A.D.) and its relationship to democratic ideals and individual differences in learning, onto the (European) Renaissance[1] that led to the birth of the humanist movement and a focus on the developmental nature of human psychology, culminating with the beginnings of modern educational psychology in the foundational writings of Johann Friedrich Herbart (1776-1841), the pragmatists William James (1842-1910), G. Stanley Hall (1844-1924), John Dewey (1859-1952), and finally Edward Lee Thorndike (1874-1949). This history is absorbing reading and essential to correct the erroneous notion of attributing the birth of mathematics education in the experimental psychology and behaviorism of Thorndike. As Berliner points out, the early forerunners of psychology pointed to the dangers of over-simplifying the study of the human mind/behavior into operational and quantitatively measurable constructs. He writes:

> What James wanted was for psychologists to remember that there were other legitimate ways to conduct inquiries about human consciousness and behavior than those that became favored by the behaviorists. ... James was quite catholic (*and pragmatic*) in his views of science, whereas Thorndike, his student was not. [Italics added] (Berliner, as cited in Alexander & Winne, 2006, HEPs, p. 9)

Calfee in chapter two fast-forwards into problems of relevance for educational psychology in the 21st century with a reflection on changes evident since the release of the first edition in 1986. Among the important questions addressed by him in this chapter are (a) what is the identity of the field (who are we?), (b) what are the really important problems (RIP) confronting the field, and (c) the question of preparing the next generation of professionals (in Ed Psych) by reflecting on the challenge posed by combining a very broad field like education with that of an equally broad field like psychology. He comments that there are too many *big* ideas of relevance in both fields which complicates the issue of the preparation of future researchers in the field. Calfee calls for a broadening of horizons for future generations of researchers with a wider exposure to theories and methodologies, instead of the traditional approach of introducing researchers to particular theories that jive with specialized quantitative methodologies that restrict communication among researchers within the field. Although he is pragmatic about integrating competing methodologies, the point of his chapter is to make an argument to broaden the training of future researchers in addition to the types of problems the field considers important to solve. Calfee also laments on the lack of interest in the field in learning and equity dimensions and concludes the chapter with a remark that is applicable to mathematics education:

> The next 20 years will pass in the blink of an eye. Barriers to fundamental change appear substantial, but the potential is intriguing. Technology brings the sparkle of innovation and opportunity but more significant are the social dimensions – the RIPs mentioned earlier are grounded in the quest of equity and social justice, ethical dimensions perhaps voiced infrequently but fundamental to the disciple. Perhaps HBEPIII (*the third edition of the handbook*) will contain an entry for the topic. (Calfee, as cited Alexander & Winne, 2006, HEPs, pp. 39-40)

The third chapter by Eric Bredo takes on a philosophical view of the nature of answers given by educational psychology. The chapter is appropriately titled "Conceptual Confusion and Educational Psychology" and begins with Ralph Waldo Emerson's story of "Man" being subdivided by the gods into "men" for the sake of dividing labor. The consequence of this division results a society which reduces "men's" status via suffered amputations (to denote sub-specialties within psychology). The message of this chapter is that human behavior when reduced or fragmented into pieces, results in findings that often do not give a *bigger* or full picture of what the findings mean in relation to other sub-specialties. Bredo examines psychology's attempt to understand or model the human mind through behaviorist psychology (stimulus-response), personality psychology (trait-treatment), and computational theories of the mind (mind-matter). His conclusions bring to mind (pun intended) the words of the early Gestalt psychologists that reducing a whole into myriad parts, analyzing each part separately, and piecing it back together never results in the whole, meaning that some essence of the human experience gets lost in the process of reduction and re-consolidation.

An emergent and oft discussed theme in HEPs is the limitation of the dominant paradigms and methods of inquiry, particularly operational constructs and statistical techniques that fail to fully capture the workings of the mind. However, none of the theorists in HEPs are willing to explicitly define what thinking is? Or what consciousness is? Or why competing paradigms in educational psychology are dichotomous and/or limited? To this end, I will attempt to answer these questions based on my work in the philosophy of science (Sriraman & Benesch, 2005). One might define human consciousness as the possibility of attending/intending, and describe specific experiences and their interpretations as possibilities for consciousness as attentions and intentions. Experiencing is therefore a synthesis of *of* and *for*. We encounter an excellent example of this synthesis when we seek to explain and/or define *self* or *world*. Any explanation, interpretation, definition, and so forth is an attending/intending flow with at least five aspects.

1. The "observer, interpreter, explainer;"
2. The "interpreted, observed, explained" or experienced object which is the context to which the interpreter refers;
3. The process of "interpreting, observing, explaining;"
4. The "interpretation, observation, explanation" that emerges from items 1 – 3; and
5. The "awareness" of and ability to distinguish the preceding four aspects of this continuum and to focus upon them individually and collectively, assigning each significance and value.

It is within this fifth aspect that perspectives occur on the other four and upon number five itself. Every aspect of this continuum provides a vast number of possibilities for consciousness, while consciousness as the possibility of the totality is not reducible to any particular aspect, and is the source most clearly reflected in the fifth aspect. This five-aspect continuum is implicit in all subject-object-process language-understanding relationships. The challenge is to preserve the totality of "consciousness as possibility" while utilizing and/or emphasizing particular aspects within it as possibilities for consciousness. Otherwise, we confuse the aspect with the whole or perhaps adopt the illusory "perspective of no perspective" – it is the processing of "consciousness as possibility" that is the source of exploring, explaining, defining – the possibility for theorizing, theologizing, biologizing, cosmologizing, psychologizing. It is the processing of "consciousness as possibility" that discusses the "possibilities for consciousness" in the contexts of the sciences, arts, and humanities (Sriraman & Benesch, 2005, pp. 44-45).

Part II of HEPs synthesizes the *entire body* of research on developmental and individual differences in academic pathways in early childhood, developmental issues in adolescence onto adult development. The other chapters in this part examine individual differences in cognitive function, models of personality, and affect and profiling of children with special needs to differentiate their schooling experiences. To me the chapter on individual differences in cognitive functioning (Ackerman & Lohman) was particularly interesting and relevant for mathematics education because it addressed theories that have been overlooked by the mathematics education community. Ackerman and Lohman include a comprehensive survey of the prominent and most validated theory of cognitive abilities which integrates prior work in other areas of psychology, namely the Catell-Horn-Caroll theory (CHC), and two non-CHC theories of cognitive abilities, namely Howard Gardner's theory of multiple intelligences and Robert Sternberg's triarchic theory of intelligence. The reader is left to resolve the dichotomy in these three very different theories. More importantly these authors tackle the controversial construct of general intelligence (simply known a "g" in psychometric parlances) and the limitations of psychometric testing in relation to gender and minority groups as well as age differences. The construct of intelligence in general and mathematical intelligence in particular have been topics of great controversy since the advent of psychometric testing. For example most modern day intelligence tests which have evolved out of the original Binet-Simon test and the Stanford-Binet test developed by Lewis Terman consist of subtests which measure numerical reasoning, digit memory, letter-number sequencing, digit symbol-coding, picture completion, block design, matrix reasoning, symbol and object assembly. In other words logical, quantitative, and visual-spatial reasoning play a significant role in IQ tests. High scores on the Stanford-Binet have been traditionally used as an indicator of giftedness and a predictor of academic success in school and beyond. Similarly psychometric batteries such as the SAT, ACT, and GRE consist of a mathematics portion which claim to predict academic success in college. However this view of intelligence has been criticized as being problematic as the items do not take into consideration socio-cultural and environmental variables that can influence performance particular among minorities and non-native English speakers (Sriraman, 2009, in press). In the studies conducted in the domain of mathematics education, mathematical intelligence in an individual has been defined in terms of: (a) the ability to abstract, generalize, and discern mathematical structures; (b) data management; (c) ability to master principles of logical thinking and inference; (d) analogical, heuristic thinking and posing related problems; (e) flexibility and reversibility of mathematical operations; (f) ability to discover mathematical principles; (g) decision making abilities in problem solving situations; (h) the ability to visualize problems and/or relations; and (i) distinguish between empirical and theoretical principles. Although these nine components of mathematical ability are applicable to students schooled in traditional mathematics, the definition is very narrow in its scope and excludes a significant proportion of the world's population whose knowledge of mathematics has a cultural basis. In socio-cultural frameworks for mathematics such as that proposed by Alan Bishop (1988) mathematical intelligence is viewed as being engaged in and aware of the six pan-cultural human activities which are (a) playing, (b) designing, (c) locating, (d) explaining, (e) counting, and (f) measuring.

Parts III, IV and V of HEPs delves into the areas of cognition, motivation, and educational content respectively. Educational content encompasses teaching and learning issues in the areas of Reading, Writing, Mathematics, Science, Social Studies, and Second Language. The chapter representing mathematics is written by Schoenfeld, entitled "Mathematics Teaching and Learning." Schoenfeld begins with the disclaimer that he has chosen to focus on areas that he considers have progressed, namely teacher knowledge, professional development, curriculum development, implementation and assessment, contextual learning, and the issues of equity and diversity. My first reading through the chapter left me a little perplexed because it ignored several genres of well researched domains within mathematics education such as the Rational Number Project, Proof; Problem Solving, Modeling and so forth, several of which he himself was a part of. I had the opportunity to ask him the reason for these omissions at a meeting in Oberwolfach, Germany in November 2007. His reasoning was to give the audience of the HEPs, namely educational psychologists the big picture of the field directly related to issues that were happening on a scale that policy was being influenced by such work. He pointed that he had referenced Frank Lester's SHBMTL in the footnote on the first page for those interested in specific research on mathematical learning. The section in his article that caught my attention was on "Equity and Diversity in Mathematics Education." In this section he brings up the issue of achievement gaps in relation to poverty, socio-economic class, and marginalized minorities in the United States. The research of Bob Moses, Danny Martin, Rico Gutstein and colleagues is used to present the relevance of the social justice angle to mathematics education. Schoenfeld concludes the chapter with optimism for research in mathematics education viewed in the long run. Part VI of HEPs presents societal and cultural perspectives in educational psychology in well written chapters that present the background history of socio-cultural frameworks in education going back to the Age of Enlightenment, the limitations of the Kantian worldview, and the place of constructivist debates within the larger historical picture. This chapter is important reading for graduate students in mathematics education, some of whom have surprised me in social situations with comments such as "Constructivism was invented in the 1980s by Ernst von Glasersfeld." The other chapters in part VI summarize educational psychology research on ethnicity and learning, the gender gaps in learning (addressed in the next section of the review), social bonds, and cultural conceptions of learning and development. Part VII of HEPs is concerned with issues of research, teaching and learning in varying classroom environments. Part VIII of the HEPs is concerned exclusively with Assessment.

Zooming in on Educational Psychology – The Domain of Mathematical Cognition

The *Handbook of Mathematical Cognition* (HoMC) is an interesting collection of essays on the cognitive and neurological processes that form the core of mathematical ability, particularly numerical ability with 27 chapters primarily concerned with the notion of number. These chapters address the developmental and biological bases of number concepts and processes, including eclectic studies which investigate causes for mathematical disabilities (developmental dyscalculia) and similarities between humans and animals in numerical processes. I became interested in the contents of HoMC when researching recent literature on gender stereotypes in mathematics, which resulted in a reference to chapter 14 in HoMC by Talia Ben-Zeev et al. on "Stereotypes and Math Performance." This particular chapter offers a survey of the major work in the area of gender stereotyping and thankfully dispels the biological explanations for gender differences such as those drawn from Camilla Benbow and Julian Stanley's studies with mathematically gifted adolescents. Ben-Zeev et al. write:

> This reasoning is based on the following logic: because this sample of gifted males and females had taken the same math classes and

> thus shared the same environmental influences, the differences must be biological. This argument is severely flawed. Research has shown that not all individuals experience the same environment in identical ways, especially when individuals are stigmatized in a given domain. (Ben-Zeev et al., as cited in Campbell, 2004, HoMC, pp. 235-236)

This chapter identifies the crucial construct of "stereotype threat effect" (also addressed in HEPs by Ackermen & Lohman), which is the phenomenon of high achieving individuals becoming targets of stereotypes which allege inferiority and reminders of the possibility of stereotype confirmation. The authors point out this phenomenon in other minority and historically stigmatized groups and other subtle ways in which stereotype threat triggers. The chapter concludes with recommendations for combating this effect based on the empirical findings of research on performance expectancy, evaluation apprehension, self-handicapping, stereotype suppression, anxiety, and physiological arousal to turn situations in which students are threatened into situations in which they are challenged to perform to their full potential (Campbell, 2004, HoMC, p. 246). Readers familiar with the book entitled *Cognitive Science and Mathematics Education* (Schoenfeld, 1987), published nearly 20 years ago by LEA, will find that this HoMC has new research based perspectives from cognitive science for the learning of arithmetic, magnitude, ratio and proportion, estimation skills, and problem solving, all important areas of concern in mathematics education. HoMC is thematically divided into five parts. Part 1 consisting of seven chapters deals with cognitive representations for numbers and mathematics. Chapter one entitled "About Numerical Representations: Insights from Neuropsychological, Experimental and Developmental Studies" (Fayol & Seron) examines among other things, pre-verbal representations of number concepts in animals and infants. The authors write:

> Both newborns and animals seem to be able to mobilize two different systems for the processing of quantities. One of these is precise and is limited by its absolute set size ... the other is extensible to very large quantities, operates on continuous dimensions, and yields an approximate evaluation in accordance to Weber's law. (Fayon & Seron, as cited in Campbell, 2004, HoMC, p. 5)

The chapter then addresses non-linguistic processing of "numerosity" in adults, relations between the Indo-Arabic code, verbal codes, and semantic representations. The authors point to the fact that many open questions remain on the nature of representations underlying arithmetic and number cognition. In Chapter two, Brysbaert reviews and discusses the findings of major studies on number recognition. In Chapter three, the subsection on developmental and cultural determinant in spatial representations is particularly interesting. Fias and Fischer write:

> What determines the left-right orientation of the mental number line? ... Western participants in number studies typically read from left to right, and this cognitive strategy may transfer from the domain of letter, word, and sentence processing to the processing of digits, numbers, and equations. (Fias & Fischer, as cited in Campbell, 2004, HoMC, p. 51)

Chapter seven by Nunez and Lakoff on the cognitive foundations of mathematics is familiar territory to mathematics education researchers. In this chapter the authors present the main theses from their book *Where Mathematics Comes From*, namely the role of conceptual metaphors in the structuring and organizing of mathematical ideas.

Part two of the HoMC, consists of seven chapters and focuses on "Learning and Development of Numerical Skills." These chapters are very relevant to the mathematics education community. In Chapter eight, "The Young Numerical Mind: When Does It Count?" Cordes and Gelman offer a provocative analysis of differing theoretical perspectives on the development of counting skills in relationship to the structure of arithmetic. These authors highlight the role of non-verbal mechanisms in young children and how it leads to the counting principle (based on cardinality).

> Paradigms in which assessments of counting are combined with its role in arithmetic are much less prevalent … Given the assumption that young children do not understand their own counting, it hardly makes sense to ask them to relate counting to mathematical operations. For example Piaget's theory is a set-theoretic one that grounds the understanding of cardinality in the operations of one-one correspondence and logical classification. For him counting in preoperational children is done by rote and without understanding … this class of accounts rejects the view that pre-schoolers have any numerical abilities. (Cordes & Gelman, as cited in Campbell, 2004, HoMC, p. 128)

The reader is presented with a very interesting accounting of the phenomenon of early counting which assumes phylogenetic and ontogenetic continuities. The authors ask the community to critically examine the nature of tasks/assessments for the study of young children's (< 3.5 years) numerical abilities to determine whether or not young children understand the notion of cardinality. There are some open questions posed by these authors which are worthwhile for researchers interested in this domain of research. In chapter ten, Miller et al., present cross-cultural insights on the nature and course of preschool mathematical development in China and the United States. Given the recent debate over Liping Ma's (1999) book *Knowing and Teaching Elementary Mathematics*, this chapter presents a complementary accounting of how early learning experiences in arithmetic in China and the United States, differences in parental beliefs and practices before formal schooling, results in significant differences on the ensuing mathematical development of children in these two countries. Part five of the book entitled "Neuropsychology of Number Processing and Calculation" with chapters by Dehaene et al. on three parietal circuits for number processing; by Butterworth on developmental dyscalculia, and Lochy et al. on rehabilitation of acquired calculation and number processing, gives the reader findings from neuropsychological research in mathematical cognition. These three chapters are tersely written and are challenging reading for one unfamiliar with this domain of research.

Zooming out of Mathematical Cognition – Beyond Educational Psychology

The *Second Handbook of Research on Mathematics Teaching and Learning* (SHBMTL) comes in two volumes which combine for a whopping total of 1324 pages. It is beyond the scope of this critical notice to review the entire Handbook, so I am choosing to focus exclusively on Part I of the book, namely "Foundations," which contains three chapters. The first chapter by Paul Cobb examines the philosophical foundations of different theoretical perspectives used in mathematics education. The second chapter by Silver and Herbst looks at role of theory in mathematics education research and the third chapter by Schoenfeld focuses on methods in mathematics education research. As an insider in the field, I naturally held both familiarity and biases on what constituted the canonical foundational literature for the field. So, I gave each one of these chapters to several colleagues who worked in the domain of psychology, and the philosophy of science, who were unaware of research in mathematics education and asked them to give me their impressions on the origins and the foundations of the field.

The two philosophers of science found the four theoretical perspectives labeled by Cobb as "experimental psychology, cognitive psychology, socio-cultural theory and distributed cognition" to cohere with the historical view of idea development and transmission to the larger community of practice in the community of scientists. The summary of cognitive psychology and sociocultural theory in Cobb's chapter overlaps with the history of educational psychology outlined in the first three chapters of HEPs. In addition Cobb outlines a design science approach to mathematics education. Design research typically involves creating opportunities for both engineering particular forms of learning and teaching and studying these forms systematically within the supportive contexts created (Cobb et al., 2003; Lesh & Clarke, 2000). Such a process usually involves a series of "iterative design cycles," in which trial outcomes are iteratively tested and revised in progressing towards the improvement of mathematics teaching and learning (English, 2007; Lesh & Sriraman, 2005). Such studies contrast with previous research involving information-processing approaches that traced the cognitive growth of individuals in selected mathematical domains such as number (e.g., Simon & Klahr, 1995). In the intervening years, design science approaches have opened up a new world of mathematics education research – we now have a greater understanding of a "learning ecology" (Cobb et al., 2003). Cobb's erudition of the usefulness and limitations of the four theoretical perspectives for the design and study of problems in mathematics education research resonate the words of the pragmatists cited by Berliner and Calfee in HEPs. Each perspective characterizes the individual in ways that are incommensurable. However Cobb argues that this is not a problematic issue when seen through the lens of the work of Richard Bernstein, a prominent 20th century pragmatist. Cobb concludes that although there are no neutral frameworks for the comparison of divergent theoretical perspectives, this does not "condemn us to absolute relativism in which theoretical decisions amount to nothing more than personal whim or taste" (p. 33). The implication of Cobb's argument for the field of mathematics education is that communication between researchers working within different paradigms is possible.

The psychologists who read Silver and Herbst's chapter reacted with shock. They found it hard to believe that our field was relatively localized in its attempts at theory generation. This chapter is a good introduction to anyone that assumes theory generation in mathematics education is a trivial matter. Silver and Herbst identify and clarify how theories that can mediate the links between research and practice. One of the implications of this chapter is that no grand theories are possible in mathematics education. Several of the ideas raised by Silver and Herbst parallel those proposed in an earlier paper by Lesh & Sriraman (2005).

The third chapter by Schoenfeld on method is structured in three parts. The first examines the relationship between theory and methodology, the second presents a framework for evaluating the quality of research, particularly the issues of trustworthiness and generalizability, and the third part applies Stokes' (1997) "Pasteur's Quadrant" to research and development in mathematics education with an emphasis on design based research.

The three chapters that constitute the foundations of mathematics education in SHBMTL are in my opinion much richer in their scope in comparison to the foundations of Educational Psychology outlined in HEPs. Even though the historical foundations of mathematics education and educational psychology show overlaps with the history of psychology, psychometric testing and roots in the philosophy of pragmatism, the diversity of the cited literature that constitutes math education foundations reveals intricate and wider connections to the disciplines outside of psychology, namely philosophy, anthropology, and sociology. Having said this, I will still re-iterate my earlier criticism that mathematics education researchers use of socio-cultural frameworks are not as historically grounded as those employed by educational psychologists researching social and cultural problems. Cobb's chapter in SHBMTL fortunately bridges this gap.

Which is the Handiest of all?

Having completed the task of reviewing the three handbooks in question, am I now bound to my promise of picking the handiest one of them all? Since the question is rhetorical in nature, I don't have to answer it. The *Handbook of Mathematical Cognition* (HoMC) is very specialized in its aims and scope and fit for those that are specifically interested in researching the mathematical cognition of number concepts. Overall, HoMC surveys the extant work on mathematical cognition from a wide variety of theoretical and research perspectives (cognitive science, computational science, and neuroscience) with emphasis on the developmental phases of numerical and mathematical ability. HoMC also sets a good example of the time, effort and teamwork needed for coherent research consolidation in particular areas of mathematical cognition, which sets an interesting precedence for consolidating the state of affairs in mathematics education research. In surveying and critiquing the state of affairs in the domain of mathematical cognition, Umland (2008) writes that “any theory of mathematical cognition and learning must ultimately articulate with our current understanding of how the mind works and with current theories of how knowledge is acquired from both an individual and social perspective (p.101).

In this respect HoMC is limited in its social perspective. The three foundational chapters of the *Second Handbook of Research on Mathematics Teaching and Learning* (SHBMTL) provide an excellent introduction to mathematics education research, and give a coherent picture of the types of frameworks, researchable questions, and methodologies available to researchers in the field. In terms of the scale of researchable problems SHBMTL understandably offers a much broader perspective than HoMC.

The *Handbook of Educational Psychology* (HEPs) is more specialized than SHBMTL, it gives the reader outside of educational psychology a peripheral but historically accurate glimpse of the world of mathematics education. Having glanced through the contents of SHBMTL, I can say that the converse is also true for a reader outside of mathematics education looking at educational psychology through the lens of this book. Of the three handbooks, HEPs is by far the most affordable. All the handbooks succeed in consolidating, filtering, and transmitting the current knowledge of the three overlapping fields of inquiry. The editors are to be commended for their efforts. HEPs in my opinion offers the broadest view of the field of education whereas the first three chapters offer of SHBMTL succeeds in show casing the much broader scale of theories and methodologies that interact with mathematics education. The issue of allowing access to these scholarly compilations to less affluent communities of scholars is one that needs to be addressed. HoMC is a very specialized book tailored towards an audience of professionals in the domain of mathematical cognition. So, I do not have any issues with the price of this particular book. However the high price of SHBMTL is perplexing given that it was sanctioned by the National Council of Teachers of Mathematics, an organization that professes the goal of equity in mathematics education. The price severely restricts access to those that cannot afford it and I have heard numerous complaints in this regard. It would serve the professed goal of equity and providing equal access to our community if we created an online server where pre-prints of all the chapters were freely available to all.

This is practiced among the community of mathematicians and it would certainly serve the goal of letting all interested parties to be privy to the numerous axiom sets that characterize our field. The field is not in want of or in search of an identity but is thriving on multiple fronts of research with multiple identities, none of which are in conflict with one another.

NOTES

1. The word Renaissance is Eurocentric in its coinage and usage and has become synonymous with the European Renaissance which began in Italy and an implied Greco-Roman historical attribution to human knowledge. Such a usage does not acknowledge the Persian Renaissance during the time of Cyrus, the Islamic Renaissance which was a necessary pre-cursor and catalyst for Europe to emerge out of the Dark ages, and the Bengal Renaissance, a social reform movement which occurred independently in Bengal at the dawn of the 18th century.

REFERENCES

Bishop, A.J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht, NL: Kluwer Academic Publishers.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

English, L. (2007). Cognitive psychology and mathematics education: Reflections on the past and the future. In B. Sriraman (Ed.), *Zoltan Paul Dienes and the dynamics of mathematical learning* (pp. 119-126). Missoula, MT: The Montana Mathematics Enthusiast, Monograph 2.

Lesh, R. & Clarke, D. (2000). Formulating operational definitions of desired outcomes of instruction in mathematics and science education. In A.E. Kelly & R.A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 113-149). Mahwah, NJ: Lawrence Erlbaum.

Lesh, R. & Sriraman, B. (2005). Mathematics education as a design science. *ZDM: The International Journal on Mathematics Education*, *37*(6), 490-505.

Lester, F.K. (2005). On the theoretical, conceptual, and philosophical foundations for research in mathematics education. *ZDM: The International Journal on Mathematics Education*, *37*(6), 457-467.

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. (1987). *Cognitive science and mathematics education.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Simon, T.J. & Klahr, D. (1995). A computational theory of children's learning about number conservation. In T.J. Simon & G.S. Halford (Eds.), *Developing cognitive competence: New approaches to process modeling*. Mahwah, NJ: Lawrence Erlbaum.

Sriraman, B. (2009, in press). Mathematical intelligence. In B. Kerr (Ed.), *Encyclopedia of giftedness, creativity and talent.* Thousand Oaks, CA: Sage Publications.

Sriraman, B. & Benesch, W. (2005). Consciousness and science. *Theology and Science*, *3*(1), 39-54.

Stokes, D. (1997). *Pasteur's quadrant*. Princeton, NJ: Brookings Institution Press.

Umland, K. (2008).A reflection on mathematical cognition: How far have we come and where are we going? *The Montana Mathematics Enthusiast*, *5*(1), 101-116.

Author's Address:
Dept. of Mathematical Sciences
The University of Montana
Missoula, MT 59812
U.S.A.
EMAIL: sriramanb@mso.umt.edu