

Mathematics and Literature

Synonyms, Antonyms or the Perfect Amalgam?

Mathematics is typically viewed as facts, procedures, and quantification and not associated with the activities of writing, discussing or reading literature. Literature on the other hand is used to expose young minds to various cultural and societal themes, historical ideas and to provide the context for critical thinking skills to develop and manifest. Mathematics and literature have one aspect in common, namely critical thinking, the process of making unbiased valid inferences. The preceding three sentences begs the question as to why mathematics and literature are regarded as antonyms when critical thinking is an attribute that is common to both fields. Can literature be used as a context for critical thinking as well as for introducing new mathematical ideas? The author explores this question in this paper by reflecting on the outcomes of using *Flatland* in a beginning Algebra course with 13–14 year-old students.

Introduction

The 21st century student lives in an age where there is a plethora of readily available information about every conceivable or imaginable thing. For instance, the Internet is one of the gateways to a universe of information unimaginable just two decades ago. Even the technologically 'unsavvy' cannot escape the barrage of information from the various media sources. It is reasonable to say that information saturation defines the human condition in the developed world. This leads to the ques-

tion of our role as present day educators. The typical school curriculum is administered in discrete packages to the students, maths, social studies, science, literature, languages, etc. Yet reality does not function in this discrete manner, which leads to the following questions:

1. What are generic skills that cut across disciplines and are valuable in the real world?
2. Is there a pedagogical model available that can implement the cultivation of these generic skills?

Reflecting on these questions led to some insights. It is safe to say that problem solving and critical thinking are generic 'thinking skills' that cut across a variety of disciplines. The mathematics education literature shows successful experiments by teacher educators at the university level using problem solving as a means to create meaningful learning experiences which students connect to personal metaphors such as 'stories' or 'games' and 'adventures' (Chapman, 1997). Other approaches used by university educators involved creating problem-solving situations to enable students 'to make isomorphisms between different problems to help them make convincing arguments to justify their ideas and promote critical thinking' (Glass, 2002, p. 75).

The development of critical thinking skills is traditionally one of the goals the liberal arts. Critical thinking is valuable not only to leaders who have to make difficult decisions but also used by business-people, historians, mathematicians, philosophers and scientists.

Critical thinking has historically found expression in mathematics in the context of connecting ontological questions about the nature of mathematics to questions in theology. To expect pre-teenagers to contemplate like professional philosophers and theologians about ontological questions is unreasonable and 'far out'! Any well-intentioned teacher attempting to introduce critical thinking by synthesising mathematics and theology can expect a bombardment of phone calls from irate parents as well as grief from administrative supervisors about the 'questionable proceedings' in the classroom. How can critical thinking be implemented in a mathematics classroom with 13–14 year olds? This was the burning question confronting the author.

The Archimedean 'eureka' moment manifested with the rediscovery of a precious book while organising a cluttered bookshelf. The answer sought came in *Flatland*, a 19th century underground publication. The author of this book, Edwin Abbott (1984), spins a satire about Victorian society in England by creating an isomorphic world called *Flatland* whose inhabitants are a hierarchy of geometric shapes and exhibit the many peculiarities of 19th century England.

The back cover of the book gives the following précis:

Flatland takes us on a mind-expanding journey into a different world to give us a new vision of our own. A. Square, the slightly befuddled narrator, is born into a place, which is limited to two dimensions — irrevocably flat — and peopled by a hierarchy of geometrical forms. In a Gulliver-like tour of his bizarre homeland, A. Square spins a fascinating tale of domestic drama and political turmoil, from sex among consenting triangles to the intentional subjugation of Flatland's females. He tells of visits to Lineland, the world of one dimension, and Pointland, the world of no dimension.

The novel culminates tragically with A. Square committing heresy by preaching about higher dimensions such as the third and fourth dimensions. The personal choice of this book as 'suitable' was reinforced with the release of the book *The Annotated Flatland*

(Abbot & Stewart, 2002) three months after the experiment was conducted. Renz (2002) in his review of the annotated book in the *Scientific American*, wrote that the ideas from *Flatland* 'anticipated ideas of today's string theory, one approach to fundamental particles. In addition to our three regular space dimensions plus time, this (string) theory postulates at least six more space dimensions curled so small as to be undetectable by current means!' (p.90)

In hindsight this book seemed to be the ideal choice to implement the goal of cultivating critical thinking and introducing abstract mathematical ideas in a mathematics classroom. After working out the logistics with the school principal, a classroom set of *Flatland* was ordered for the Algebra course taught by the author, a full time mathematics teacher, at a rural mid-western public high school in the United States. One can imagine the initial reaction of the class with twenty-three students when copies of the book were handed out:

I thought this was Algebra class...why are we reading a book?

This book is sexist... it talks about subjugating females.

The author set up this classroom experiment by explaining that the book was a work of fiction and would add some 'spice' to the math class. Since the class met every day for fifty minutes, Thursdays were set aside for the book, and a portion of Friday was used to review, reinforce, and expand the mathematical ideas connected to the reading. A reading schedule was provided to the class. The book was read over four weeks in November 2001. The fact that reading, writing and discussing the book replaced one of the tests in the class went over very well with the students. The remainder of the paper is devoted to presenting the classroom discussions generating from the book and to make the argument that mathematics fiction successfully 'marries' literature and mathematics and creates the ideal forum to nurture critical thinking as well as introduce sophisticated mathematical ideas to 13–14 year old students.

Week 1: Social distinctions, polygons and limits

The class was abuzz on the day the first discussion of the book was scheduled. Several students were explaining to others how to interpret the workings of *Flatland*. Several other students were engaged in a lively debate about the status of the working class and women in *Flatland*. These were good indications that the book was a good choice. Each discussion day began with students' 'free' writing for ten minutes about their thoughts on the reading and questions that were provoked by the book. The class was then split into six groups. Each groups' task was to reach a consensus on one question appropriate for discussion. These questions were used to moderate the classroom discussion. The questions this week related to understanding the hierarchy of shapes present in *Flatland* and the strange methods used by the inhabitants to distinguish one another. The discussion on various polygons was a thorough review of geometry covered in the earlier years. One student made the point:

The lines in *Flatland* are not lines... they are actually very thin isosceles triangles. Imagine taking a big isosceles triangle and squeezing the base, making it narrower and narrower... it ends up looking like a line.

This comment created the perfect opportunity to introduce the analogous notion of a limiting process in mathematics. Since circles in *Flatland* were defined as regular n -gons with large values of ' n ', the author used this to introduce Archimedean constructions to create better and better approximations to the value of π .

On a non-mathematical note, several students remarked that the seemingly strange methods used by the inhabitants of *Flatland* paralleled the social structure of the school and society in general. This was a manifestation of critical thinking.

The way they 'feel' each other to discover what class they are in is what we do now with appearances. If someone looks a certain way we will not associate with them... the emphasis is put on the appearance.

The fair abundance of 10° specimens, absolutely destitute of civic rights parallels our society treats poor people with no education with no regard and abuses their rights, like the treatment of the migrant workers.

Week 2: Comparing and contrasting Flatland and society today

The discussion from the previous week was fresh in the minds of the students and five out of the six group questions pertained to the living conditions of the inhabitants of *Flatland*, the structure of their society and the position of women in society. This class session was marked by a lively debate as summarised by the various contrasting viewpoints in the following classroom vignettes.

It [*Flatland*] is a lot like today because the lower classes get a poorer education, little respect, and do most of the work. The higher classes get a good education, therefore they can use this to gain respect and higher standings. This is true of our society today because the lower classes also go to poorer public schools where they don't learn as much because of lack of facilities... so they don't go as far in life and can never make it to the higher classes. The higher class continues to climb and gain more knowledge, power and money by getting a better education at private expensive schools.

The upper class in both worlds think they are the best. In today's society... on paper the rich do not have more rights but it seems like they do. If the upper class break laws they have to face the same consequences... although they may be able to afford better



lawyers and get away with some loophole or the other.

There are different classes in our society but they are not set apart by rules, they are equal and have the same opportunities and freedoms.

In today's society we do not consider our priests the highest class although it used to be that way a long time ago. Also women and men are considered equal by our society, it is not so in Flatland.

Fashion as in the 'swaying of the back' in public is parallel to how fashion plays out in society now. People have to 'look' a certain way in public now. Even today in our society woman are looked at as the weaker gender.

Flatland is a very odd place to live. The people aren't racist... they are 'shapist', discriminating on the basis of shape. As a girl I would feel discriminated against, having to keep moving constantly according to the laws of Flatland. Our school is a lot like Flatland because many girls think they need to dress or act a certain way in order to get accepted. So our society may not be racist but it sure is 'shapist'.

Week 3: Dimensions and dilemmas

The third week was characterised by a discussion of the notion of dimension. Just as A. Square, the protagonist from Flatland somehow realised that higher dimensions existed (such as our world, Spaceland), students raised the possibility of the existence of higher dimensions. Students were divided about the existence of a fourth dimension, until one of the groups mentioned that time could be considered as the fourth dimension. Another student remarked that if one thought of dimension as a mathematical concept, we could have as many dimensions as we wanted.

This discussion was the perfect stimulus to present simple notions from relativity and to introduce the model of Minkowskian space-time geometry. The students were fascinated

with the idea of objects not having length in an absolute sense and length being dependent on the observer's frame of reference.

The class pursued the analogies used in *Flatland* to illustrate higher dimensional objects existed. One group argued for the existence of four-dimensional objects as follows:

A point has no dimension. Take two points and connect with a line, that is one-dimensional. Take two lines, connect them to get a square, which is two-dimensional. Two squares give a cube, which is 3-D. Now two cubes connected should give the next level which is 4-D.

Another group made the following remark about the construction outlined in Figure 1.

The geometric sequence that results from the construction (Figure 1) 1 point (dot), 2 points (line) 4 points (square), 8 points (cube), 16 points (hyper-cube)... shows that the construction can be carried on as long as you want and shows its mathematically possible to have very high dimensions.

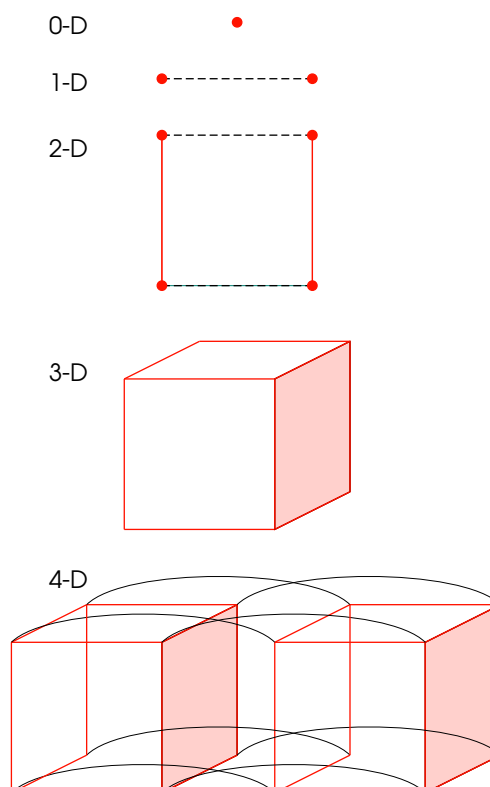


Figure 1

As indicated in Figure 1. *Flatland* brought forth a rich discussion on the idea of dimension and led the students to imagine new possibilities. The author also introduced the idea of decomposing objects into a certain number of self-similar pieces to produce 'fractal' dimensions from fractal geometry. The analogy of moving 'up' dimensions by using self-similar objects made the students open to moving 'between' dimensions when constructing fractals. One student remarked, 'The Sierpinski triangle is a figure between a line and a triangle because it is all broken up, so it makes sense that it has a dimension between 1 and 2'. This is a very sophisticated mathematical idea being expressed by this student. Some other students tried to draw a distinction between 'real' dimensions versus 'mathematical' dimensions by trying to connect the notion of dimension to the degree of polynomials. Some students who were fans of the TV series *Star Trek* were very imaginative and remarked.

It's really easy to convince a spacelander (us) that a 4-D world exists. We simply have to travel through a worm hole and on the other side will be a hyper-cliff consisting of little hyper-spheres, hyper-cubes, hyper-triangles and the spacelander would then believe.'

I believe one [the fourth dimension] would exist but it would be like a person within a person

Week 4: The next frontier

The final discussion revolved around the main themes of the book. Many of the students were disappointed by the tragic ending of the book and had some startling insights to analogies in society. The questions revolved around the notion of free speech and expression and instances where people were persecuted because of their beliefs or ideas.

The discussion was 'heavy' especially when some students pointed towards examples from history that were analogous to the tragic end of A. Square.

A. Square was ahead of his time. It is a lot like the person¹ who said that the earth is not the centre of the universe and the church imprisoned him.

There was this movie² where they threw this guy in prison because he was a genius and knew every little thing about the government and all their secrets.

...Like Stalin started getting rid of all these intellectuals because they were writing against communism

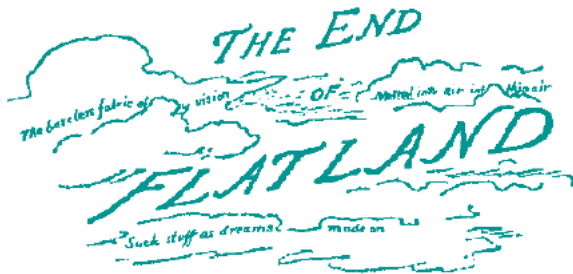
Conclusions and implications

The process of critical thinking could be viewed as the use of reasoning in the pursuit of 'truth'. Critical thinking has also been defined as the ability and tendency to gather, evaluate, and use information effectively (Beyer, 1985). The students in this Algebra course demonstrated their capacity for critical thinking by making inferences on 'truths' about society and life as is evidenced in the various classroom vignettes. The book became a didactic tool of sorts and provided the ideal scaffolding for students to critically examine the norms and biases of society. The students were reasoning by analogy, as well as using their imagination. The ideas in *Flatland* also exposed students to some very advanced mathematical ideas such as dimension. This created the perfect setting for the teacher (author) to expose students to broader mathematical notions such as limits, non-Euclidean geometries such as the Minkowskian space-time geometry and Fractal geometry, and historical techniques of approximation. Although the students were discussing the novel, there was a general inquisitiveness about a variety of societal issues and new mathematical ideas. Students

1. The student was referring to Copernicus.

2. There is a whole genre of movies where highly intelligent people who stumble upon classified information are imprisoned under the pretext of 'national security'.

were willing to listen to alternative opinions and most importantly were willing to examine their own biases and prejudices (Facione, 1990). The experiment with *Flatland* was talked about for the rest of the school year and some of the students suggested in reading one of the many sequels to the book. This was done in the second semester but space constraints prevent another exposition at this juncture. Perhaps a sequel will follow!



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