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Does mathematics gifted education need a working philosophy of creativity?

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ABSTRACT: *In this paper, we present existing views and approaches to creativity with emphasis on their links with mathematics. Educational and social views of creativity in general, and mathematical creativity in particular is discussed. While institutional and societal indifference to the needs of the mathematically gifted is a well known phenomena, recent research studies reveal that even less attention is paid to the development and nurturing of creativity in mathematically gifted. We will discuss the need of such particular attention form the mathematician's, psychologists' and educators' points of view. We will conclude our discussion with recommendations for the kind of learning and teaching environment for mathematically gifted students in order to stimulate and nurture their creativity as well as to benefit the needs of other students in that setting.*

Keywords: inclusive schooling; culture; theories of creativity; theories of giftedness; teaching and learning environments; Russian mathematical circles

1. INTRODUCTION

The constructs of mathematical giftedness and mathematical creativity are inter-connected, with creativity implicitly implying giftedness (Sriraman, 2005). However studying a mathematician's or a student's creativity is a very difficult enterprise because most traditional operationalized instruments fail to capture extra cognitive traits such as beliefs, aesthetics, intuitions, intellectual values, self imposed subjective norms, spontaneity, perseverance standards, and chance which contribute towards astonishing acts and products of creative endeavors (Shavinina & Ferrari, 2004).

2. A BRIEF OVERVIEW OF LITERATURE ON CREATIVITY

2.1 Sternberg's classification of approaches to the study of creativity

The Handbook of Creativity (Sternberg, 2000), which contains a comprehensive review of all research available in the field of creativity suggests that most the approaches used in the study of creativity can be subsumed under six categories, which are: mystical, pragmatic, psychodynamic, psychometric, cognitive and social-personality. We will briefly review each of these approaches for the reader unfamiliar with these approaches.

The Mystical Approach

The mystical approach to studying creativity suggests that creativity is the result of divine inspiration, or is a spiritual process. In the history of mathematics, Blaise Pascal claimed that many of his mathematical insights came directly from God. The renowned 19th century algebraist Leopold Kronecker said that "God made the integers; all the rest is the work of man" (Gallian, 1994). Kronecker believed that all other numbers being the work of man were to be avoided. Although his radical beliefs did not attract many supporters, the intuitionists advocated his beliefs about constructive proofs many years after his death. There have been attempts to explore possible relationships between the mathematician's belief on the nature of mathematics and their creativity (Davis and Hersh, 1981; Hadamard, 1945; Poincaré, 1948). There is certainly a relationship between a mathematician's belief on the nature of mathematics and creativity. The neo-Platonist view is helpful to the research mathematician because of the innate belief that the sought after result/relationship already exists (Sriraman, 2004). The formalist viewpoint that mathematics is a man-made game, which is meaningless in reality, is a pessimistic point of view to the research mathematician and not helpful to sustain prolonged research, although Hersh (2006) has successfully demolished this viewpoint.

<u>The Pragmatic Approach</u>

The pragmatic approach entails "being concerned primarily with developing creativity" (Sternberg, 2000, p.5), as opposed to understanding it. Polya's (1954) emphasis on the use of a variety of heuristics for solving mathematical problems of varying complexity is an example of a pragmatic approach. Thus, heuristics can be viewed as a decision-making mechanism, which lead the mathematician down a certain path, the outcome of which may or may not be fruitful. The popular technique of brainstorming used in corporations is another example of inducing creativity by seeking as many ideas or solutions possible in a non-critical setting.

<u>The Psychodynamic Approach</u>

The psychodynamic approach to studying creativity is based on the idea that creativity arises from the tension between conscious reality and unconscious drives (Hadamard, 1945; Poincaré, 1948, Sternberg, 2000, Wertheimer, 1945). The four-step gestalt model (preparation-incubation-illumination-verification) is an example of the use of a psychodynamic approach to studying creativity. As expounded in chapters1 and 2, the gestalt model has served as the kindling for many contemporary problem solving models (Polya, 1954; Schoenfeld ,1985; Lester, 1985). Early psychodynamic approaches to creativity were used to construct case studies of eminent creators such as Albert Einstein, but this approach was criticized by the behaviorists because of the difficulty of measuring proposed theoretical constructs.

The Psychometric Approach

The psychometric approach to studying creativity entails quantifying the notion of creativity with the aid of paper and pencil tasks. An example of this would be the Torrance Tests of Creative Thinking developed by Torrance (1974), which are used by many gifted programs in middle and high schools, to identify students that are gifted/creative. This test consists of several verbal and figural tasks that call for problem-solving skills and divergent thinking. The test is scored for fluency, flexibility, originality (the statistical rarity of a response) and elaboration (Sternberg, 2000). In his review, Sternberg (2000) analyses positive and negative sides of such approach. On the positive side, these tests allow for research with non-eminent people, are easy to administer, and objectively scored. The negative side is that numerical scores fail to capture the concept of creativity because they are based on brief paper and pencil tests. Researchers call for use of more significant productions such as writing samples, drawings, etc to be subjectively evaluated by a panel of experts instead of simply relying on a numerical measure.

The Cognitive Approach

The cognitive approach to the study of creativity focuses on understanding the "mental representations and processes underlying human thought" (Sternberg, 2000, p.7). Weisberg (1993) suggests that creativity entails the use of ordinary cognitive processes and results in original and extraordinary products. These products are the result of cognitive processes acting on the knowledge already stored in the memory of the individual. There is a significant amount of literature in the area of information-processing (Birkhoff, 1969; Minsky, 1985) that attempts to isolate and explain cognitive processes in terms of machine metaphors. In mathematics Boolean Algebra was an attempt to formalize the laws of thought and considered a major breakthrough in mathematical psychology…as the first step in symbolic logic towards the mechanization of mathematical thinking (Birkhoff, 1969).

The Social-Personality Approach

The social-personality approach to studying creativity focuses on personality and motivational variables as well as the socio-cultural environment as sources of creativity. Sternberg (2000) states that numerous studies conducted at the societal level, indicate that "eminent levels of creativity over large spans of time are statistically linked to variables such as cultural diversity, war, availability of role models, availability of financial support, and competitors in a domain." (p.9)

Most of the recent literature on creativity (Csikszentmihalyi, 1988, 2000; Gruber & Wallace, 2000; Sternberg & Lubart, 1996) suggests that creativity is the result of confluence of factors from the six aforementioned categories. The "confluence" approach to the study of creativity has gained credibility and the research literature has numerous confluence theories for better understanding the process of creativity. Hence we will now review three of the most commonly cited confluence theories of creativity, which are the "systems approach" (Csikszentmihalyi, 1988, 2000); "the case study as evolving systems approach" (Gruber& Wallace, 2000), and finally the "investment theory approach" (Sternberg & Lubart, 1996).

2.2 Confluence Theories of Creativity

Csikszentmihalyi's Systems Approach

Csikszentmihalyi's (2000) systems approach (see Figure 1) takes into account the social and cultural dimensions of creativity, instead of simply viewing creativity as an individualistic psychological process. The system approach of Csikzentmihalyi (1988, 2000) studies the interaction between the individual, domain and field. The field consists of people who have influence over a domain. For example, editors of mathematics education research journals would have influence on the domain of mathematics education. The domain is in a sense a cultural organism that preserves and transmits creative products to other individuals in the field.

Csikszentmihalyi's (2000) systems model (see Fig. 1) suggests that creativity is a process that is observable at the "intersection where individuals, domains and fields interact" (p.314). Csikszentmihalyi (2000) claims that the three components, namely, individual, domain and field are necessary by arguing as follows. One cannot simply focus on the individual aspects of creativity because the individual operates in a cultural or symbolic (domain) aspect as well as a social (field) aspect. "The domain is a necessary component of creativity because it is impossible to introduce a variation without reference to an existing pattern. New is meaningful only in reference to the old"(Csikszentmihalyi, 2000, p.314-315). Thus creativity occurs when an individual makes a change in a given domain, and this change is transmitted through time. The personal background of individuals and their position in a domain naturally influence the likelihood of their contribution. For example, a mathematician working immersed in the culture of a research university is more likely to produce research papers because of the time available for "thinking" as well as being immersed in a culture where ideas flourish. It is no coincidence that in the history of science, there are significant contributions from clergymen such as Pascal, Copernicus and Mendel, to name a few, because they had the means and the leisure to "think". Csikszentmihalyi (2000) then argues that novel ideas that result in significant changes are unlikely to be adopted unless they are sanctioned by a group of experts that decide what gets included in the domain. These "gatekeepers" (experts) constitute the field. For example, in mathematics, the opinion of a very small number of leading researchers was enough to certify the validity of Andrew Wiles's proof to Fermat's Last Theorem.

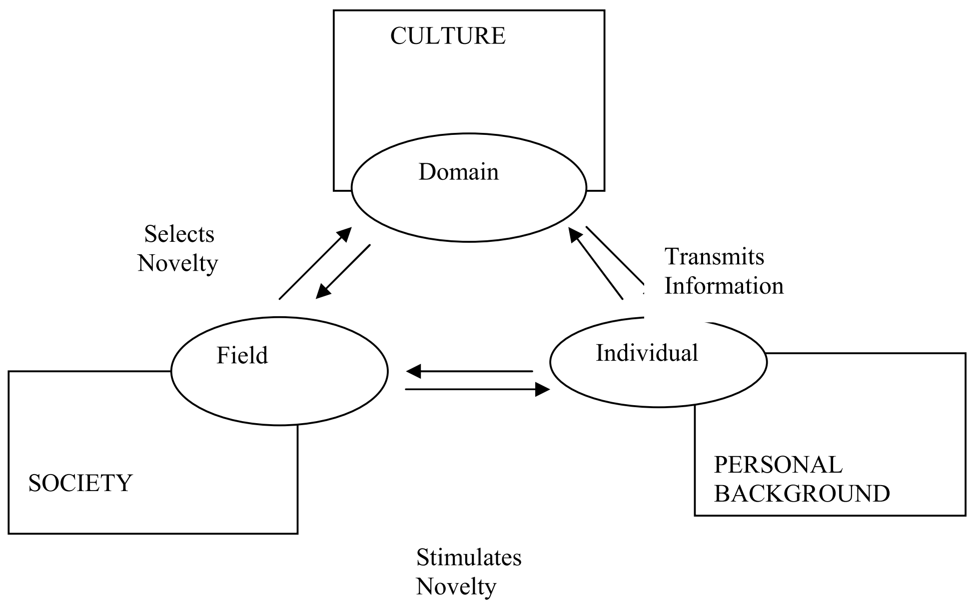

Figure 1. Csikszentmihalyi's (2000) Systems Model

So, in summary, the systems model of creativity suggests that for creativity to occur a set of rules and practices must be transmitted from the domain to the individual. The individual, then must produce a novel variation in the content of the domain, and this variation must then be selected by the field for inclusion in the domain (Csikszentmihalyi, 2000, p.315). Thus, Csikszentmihalyi's (2000) contends creativity research should not focus excessively on eminent individuals, but also focus on communities that potentially contain highly gifted individuals.

Gruber & Wallace's Case Study as Evolving Systems Approach

In contrast to Csikszentmihalyi's (2000) argument that calls for focus on communities in which creativity manifests, Gruber & Wallace (2000) propose a model that treats each individual as a unique evolving system of creativity and ideas, and therefore each individual's creative work must be studied on their own. This viewpoint of Gruber & Wallace (2000) is a belated victory of sorts for the Gestaltists, who essentially proclaimed the same thing almost a century ago. Gruber & Wallace's (2000) use of terminology that jives with current trends in psychology seems to make their ideas more acceptable. They propose a model that calls for "detailed analytic and sometimes narrative descriptions of each case and efforts to understand each case as a unique functioning system (Gruber&Wallace, 2000, p.93). It is important to note that the emphasis of this model is not to explain the origins of creativity, nor is it the personality of the creative individual, but on "how creative work works?" (p.94). The questions of concern to Gruber & Wallace are:

1) What do creative people do when they are being creative? and 2) how do creative people deploy available resources to accomplish something unique? In this model creative work is defined as one that is novel and has value. This definition is consistent with that used by current researchers in creativity (Csikszentmihalyi, 2000; Sternberg & Lubart, 2000). Gruber & Wallace (2000) also claim that creative work is always the result of purposeful behavior and that creative work is usually a long undertaking "reckoned in months, years and decades"(p.94). The researcher/teacher does not agree with the claim that creative work is always the result of purposeful behavior. One counterexample that comes to mind is the discovery of penicillin. The discovery of penicillin could be attributed purely to chance. On the other hand there are numerous examples that support the claim that creative work sometimes entails work that spans years. In mathematical folklore there are numerous examples of such creative work. For example, Kepler's laws of planetary motion were the result of twenty years of numerical calculations. Andrew Wiles's proof of Fermat's Last Theorem was a seven year undertaking. The Riemann hypothesis states that the roots of the zeta function (complex numbers z, at which the zeta function equals zero) lie on the line parallel to the imaginary axis and half a unit to the right of it. This is perhaps the most outstanding unsolved problem in mathematics with numerous implications. The analyst Levinson undertook a determined calculation on his deathbed that increased the credibility of the Riemann-hypothesis. This is another example of creative work that falls within Wallace & Gruber's (2000) model.

The case study as evolving system model has the following components to it. First, it views creative work as multi-faceted. So, in constructing a case study of a creative work, one has to distill out the facets that are relevant and construct the case study based on the chosen facets. Wallace & Gruber's (2000) model propose numerous facets that can be used to construct an evolving system case study. These facets are: uniqueness of the work, epitome (a narrative of what the creator achieved), systems of belief (an account of the creator's belief s system), modality (whether the work is a result of visual, auditory or kinesthetic processes), multiple time-scales (construct the time-scales involved in the production of the creative work), dynamic features of the work (documenting other problems that were worked on simultaneously by the creator), problem-solving, contextual frame (family, schooling, teachers influences), and values (the creator's value system). So in summary, constructing a case study of a creative work as an evolving system entails incorporating the many facets suggested by Wallace & Gruber (2000). One could also evaluate a case study involving creative work by looking for the above mentioned facets.

3. VIEW OF CREATIVITY EXPRESSED BY SOCIAL INSTITUTIONS AND THE COMMUNITY

The literature summarized above does not take into full consideration cross-cultural differences in views of what constitutes mathematical creativity. Cultural and social aspects play a significant role in what the community, in general, and the school system, in particular, considers as "creativity" and how they deal with it.

Numerous studies (Crammond, 1994; Davis, 1997; Smith, 1966; Torrance, 1981) indicate that the behavioral traits of creative individuals very often go against the grain of acceptable behavior in the institutionalized school setting. For instance, negative behavioral traits such as indifference to class rules, display of boredom, cynicism or hyperactivity usually result in disciplinary measures as opposed to appropriate affective interventions. In the case of gifted students who 'conform' to the norm these students are often prone to hide their intellectual capacity for social reasons, and identify their academic talent as being a source of envy (Massé & Gagné, 2002). History is peppered with numerous examples of creative individuals described as "deviants' by the status quo. The stifling of creativity at the K-12 levels is often collectively rationalized under the guise of doing what is supposedly good for the majority of the students, or invoking the oft-misused term 'equity', or appealing to curricular plans and school achievement goals etc. etc. The recent passing of the No Child Left Behind Act (NCLB) in the United States under the guise of 'equity' has brought to the forefront the debate of what is to be done with creative and gifted students in the classroom. Recently Marshak (2003) wrote that the NCLB call for accountability based on standardized testing for the traditional skills of reading, writing and arithmetic valued in the industrial societal setting is a giant step **backwards** to the 1940's. Based on recent reports released by the U.S. Department of Labor, Marshak (2003) further states that besides the three "traditional" R's, numerous additional skills such as problem solving and creative thinking skills are necessary for success in the global societal setting of 21st century. Even at the tertiary levels there have been criticisms about the excessive amount of structure imposed on disciplines by academics (Creme, 2003, p.273). In summary, a significant proportion of the literature indicates that creativity is viewed as a fringe commodity, tolerated and nurtured by some teachers and typically not encouraged. However such a viewpoint may simply be a function of culture and location.

Australian researchers report a similar situation with education in general and education of mathematically gifted in particular. In their analysis of the current situation, Diezmann, & Waters (2002) state that despite all the discourses about the need of a 'clever country' and increasing value of the role of creative individuals, the situation doesn't seem to be better 100 years after it was mentioned that the country looks like 'the paradise of mediocrity and the grave of genius'. In fact, the authors refer to the Australian Senate report on education of gifted children that acknowledges that focusing on minimum standards could have a disastrous effect on satisfying the special needs of the gifted who are already affected by "underachievement, boredom, frustration, and psychological distress, (and) … negative attitudes and mistaken beliefs'. This situation is particularly dire in the case of mathematics where gifted children are affected in multiple ways by the generally negative attitude of others towards mathematics. This attitude identifies gifted children as a 'marked' group or 'deviant' population. Negative community attitudes towards the gifted reaches its extremes in derisive labelling of such children as 'little Einsteins' or 'nerds'. In such a general and pervasive anti-intellectual atmosphere that seems unfavourable to mathematically gifted students these children need particular support and resilience. Diezmann & Watters (2002) suggest that teachers need to be highly supportive of gifted students who are predisposed towards high performance and creative achievement instead of being indifferent or unaware of their needs or even react negatively to any special attention that these children might require.Based on this background of the existing situation in school systems that seems to be in accord with the public sentiment one can ask what can be done for gifted children to help them to become more creative.

In the next section, we present three particular points of view: psychological, mathematical and educational.

4. NEED FOR NURTURING AND SUPPORT OF CREATIVITY IN THE MATHEMATICALLY GIFTED

4.1 Psychologist's point of view of mathematical abilities related to creativity

In his longitudinal study on mathematical ability, Krutetskii (1976) argues that a successful mathematical activity requires particular combination of personal traits. He argues that having superior mathematical abilities does not necessarily allow the gifted individual to reach higher mathematical summits. His findings are based on the study of a group of very talented pupils of different ages, biographies of renowned mathematicians, research literature and survey distributed among practicing teachers and professional mathematicians. The paper by Karp in this issue also addresses Krututeskii's work from a contemporary viewpoint relevant for teacher education.

First, Krutetskii refers to the work of Myasishev (Мясищев) who stated that one can not become a creative mathematician without enjoying mathematical work. Joy of mathematics boosts a desire to search, mobilizes hard working habits and dynamics. Second, the personality of teachers plays an important role. Sometimes, a very able student may not show any special interest in the subject or high results. But if teacher succeeds to discover her hidden talent and boost student's interest this student can become very successful as seen in the biographies of Lobatchevskii, Ostrogradskii, Luzin and others.

The next factor revealed by Krutetskii (1976) is related to the emotional nature of mathematical activity saying that all gifted students in his study showed very high level of emotions when they succeed to solve a difficult problem or made a mathematical discovery. Also, Krutetskii (1976) emphasizes the importance of aesthetical values of mathematical work. Citing Revesh, who said that “A mathematician creates because the beauty of mental constructions brings him joy’, Krutetskii mentions that for his students a good solution made them happy in the same way as a nice combination in chess. Their entire look showed an enjoyment: their eyes were lightened; they were rubbing their hands and they called each other to share especially nice solutions.

Hard work is another quality considered by Krutetskii (1976) as crucial for creativity in mathematics. In fact, as it was mentioned by Lavrentijev quoted by Krutetskii, the main condition of mathematical creativity is the ability to work hard constantly, over a long period of time. Often, it takes months, years and even decades for mathematician to reach her goal looking constantly for a way to solve a problem, trying to find a better one among 1000 others. These features have been observed also in gifted students from Krutetsii’s experimental group.

To complete his list of personal characteristics of gifted and creative mathematicians Krutetskii (1976) brings another three factors. In fact, in order to be creative, a mathematician must also be innovative and have courage not only to create something new but at the same time destroy old established knowledge. At the same time, a creative mathematician has to be critical of her work. Especially regarding gifted students, we need to be careful of not giving them too much credit but rather educating them on the value of critical evaluation of her work and her habits. Krutetskii (1976) finishes with the importance of not focusing only on mathematics but to develop a more harmonious (well-rounded) personality in order to be creative.

4.2 Mathematician’s point of view of the role of creativity in mathematical discoveries

Miller (1997) analyses Poincaré’s conception of ‘sensible intuition’ as a process driven by the ability to perceive the whole of the argument at a glance that allows for a selection and assembling of the appropriate combination of mathematical facts. This occurs using the “unconscious” rules of aesthetics and intuition going beyond pure logic to get some sense of the steps in a mathematical proof without access to visual imagery. According to Poincaré, a ‘special aesthetic sensibility’ helps mathematicians to filter few combinations that are ‘harmonious’ and ‘beautiful’ making intuition as an ingredient of creativity. Due to this creative component, mathematicians work on their invention building a network of thought connecting elements from widely separated domains, the process which is unconscious, subtle and delicate that must be ‘felt rather than formulated’ leading to unexpected combination of mathematical facts and to scientific invention.

What conditions are to be met in order to help gifted students to be more creative? First one can learn from reflections of renowned mathematicians when they share their moments of discovery. Gnedenko (1991) analyses different situations in which he was making his own discoveries: one when he posed a new task to himself (related to the Taylor’s series) and found a solution in a different way. Another example is related to Lusin’s work on trigonometric series. Gendenko was reading an article written by another mathematician and found another solution missed by the author. The third example is related to his learning experience from Hinchin's seminars on special topics of his own research interest when Hinchin succeeded to attract his students with fresh open new problems. Gnedenko lists several conditions to foster mathematical creativity in students. These conditions are (1) creation of special research atmosphere as a source of intellectual inspiration, (2) being a part of a team that works on real novel complex problems, (3) presence of teachers whose approach is based on patient and passionate guidance with some important hints and advice and not telling students the solutions. Finally, intrinsic motivation is also very important as it helps to mobilise inner forces and passion of intellectual hard work on certain within long period of time (ability to focus and to concentrate). Seeing hard work with several iterations, sometimes failing to get immediate result are also necessary conditions for creative mathematical work. Gnedenko argues that the solution to a problem may come as a sudden 'aha' idea that might be seen as easy and effortless inner 'vision'. He refers to examples from writing of Poincaré, Gauss and Hadamard.

Gnedenko gives examples of such sudden insights that happened in his life in very different situations not related to any mathematical activity: during teaching, shopping, traveling or even during night. One example is striking: after 3 days of useless search for a solution to one problem, he told to his teacher (Hinchin) about his doubt of the correctness of a featured conjecture. Coming home, he was very excited, he could not eat, talk to people – his brain was occupied with the problem. While thinking of a problem, he fell asleep and when he woke up, the ready-to-write proof was already in his head. Analyzing the cause of this 'sudden' insight Gnedenko argued that even though he was asleep the brain continued to work on an unconscious level. But this 'work' has to be prepared by previous process (often seemingly inefficient).

In this sudden aha effect, Gnedenko sees a parallelism between mathematical creation and poetry: both mathematicians and poets are trying to catch up with a 'burn-up bird' ('жар птица'), invisible and hardly achievable and both reach it often quite suddenly but this is preceded by long thinking and searching. Finally Gnedenko mentioned that creative mathematical work has to be related to discovery of something completely novel. He cites the Russian poet V. Mayakovskii who said that the person who found for the first time that two plus two gives four by say putting two matches with two other matches is a great mathematician.

4.3 Educators' point of view on development creativity in gifted students

According to Gnedenko (1991) mathematical giftedness (математическая одаренность) is not as rare in humans as one might think. But this personal trait of creativity can appear in different ways in different people. One person could be interested in generalizing and more profound examination of already obtained results. Another person shows an ability to find new objects for study and to look for new methods in order to discover their unknown properties. The third type of person can focus on logical development of theories demonstrating extraordinary sense of awareness of logical fallacies and flaws. A fourth group of gifted individuals would be attracted to hidden links between seemingly unrelated branches of mathematics. The fifth would study historical processes of the growth of mathematical knowledge. The sixth would focus on the study of philosophical aspects of mathematics. The seventh would search for ingenious solutions of practical problems and look for new applications of mathematics. Finally, someone could be extremely creative in the popularisation of science and in teaching.

Thus, Gnedenko (1991) relates giftedness directly with creativity. He recognizes that everybody can possess a certain degree of creativity but educational systems and background (school, family, etc.) might become an obstacle to the gifted person if the surrounding system rejects novelty and discourages efforts to look at new aspects of the problem or to go beyond known facts. . The teaching approach can also lead to some obstacles for fostering of mathematical giftedness if the teacher doesn't pay enough attention to gifted students who might loose their interest in going further in their learning of mathematics.

The history of Soviet mathematics provides with a striking example of a coexistence of two different approaches to mathematics education, one embedded into the general lay public educational system implementing the blueprint based on the European concepts of the late 19th century, and the other one focusing mainly on gifted children and having flourished starting from 1950s onwards. The latter took the form of a complex network of activities including "mathematics clubs for advanced children" (Russian "кружки" (*kruzhki*), lit. "circles" or "rings", usually affiliated with schools and universities but some were also home-based), Olympiads, team mathematics competitions, (*mat-boi*, literally "mathematical fight"), extracurricular winter or summer schools for gifted children, publication of magazines on physics and mathematics for children (the most famous being the *Kvant*, lit "Quantum"), among others (Freiman & Volkov, 2004). .

All these activities were free for all participating children and were based solely on the enthusiasm of mathematics teachers or university professors. This process led to the creation of a system of formation of a "mathematical elite"[1] in the former USSR focused first and foremost on "gifted children", which was in a sharp contrast with the "egalitarian" regular state-run schools targeting "average student" and thus neglecting the needs of all those above average level. This situation was not new to the USSR's educational system but was rooted in the former tsarist Russia' regular school system which usually did not pay much attention to gifted children. Only a few gifted children such as young A.Kolmogorov were able to benefit from a unique extra-curriculum pedagogical environment allowing them to enjoy the beauty of mathematical discovery. He attended a small private school organized by his grand-mother at home for a small group of students of various ages in which the teachers used the most recent

[1] A similar process took place in other disciplines, in particular, physics, but in the present paper we will focus on mathematics education.

pedagogical innovations. A. Kolmogorov (1988) witnesses that at the age of 5 or 6 he was pleased with his discovery of the regularity of the sum of odd numbers: $1=1^2$, $1+3=2^2$, $1+3+5=3^2$, etc. The report of the Kolmogorv's 'mathematical discovery' was published in the school magazine.

An interesting episode in Kolmogorov's story is related to his further schooling at a private gymnasium organized by "radically oriented intellectuals"[2]. A mixed school (for girls and boys) with the curriculum of "boys' gymnasium" (*i.e.*, a college) gave the students an opportunity to study according to their own interests and levels (Kolmogorov could for example, take a math course from one grade higher). At the same time, students felt responsible to study hard and to get best results for the tests and state examinations. It is not surprising that a school of this kind was in contradiction with "regular" state schools and thus was under constant threat of closing by the officials.

After the revolution of 1917, the new Soviet government closed all private schools and established a completely new school system with new curriculum. The system was aimed at offering a basic education to the entire population and, at the same time, making the education more practice-oriented. As result of the implementation of these ideas, "the mathematics program lost much of its theoretical content. Pupils studied mathematical 'recipes' applicable in specific practical situations, often without consideration of their theoretical bases"[3]. It remains unclear what the situation was of gifted students in those years, yet the sources point at the lack of knowledge displayed by those who graduated from the schools based on such "innovative" approaches as the so-called "brigade-project" organization (Vogeli, 1968 with reference to Bradis, 1954, p. 38).

As a reaction to this gloomy situation, the government declared these methods as 'errors' and ordered to make necessary corrections in the school curriculum in the early 30s. Thus, "pre-Revolutionary mathematics texts were resurrected, revised, and made official standard" (Vogeli, 1968).

Resuming our analysis, we can state that keeping an explicitly egalitarian approach to education, the Soviet education system at one moment (namely, in earlier 1930s) began spending much more money and effort to identify promising individuals and to provide them with opportunities to develop their talents (Blazer, 1989).

As the officials struggled to meet the needs of growing economy and to maintain the access to schools for everybody, numerous initiatives came from prominent mathematicians and scientists. One of the striking examples was the first mathematical

[2] "Радикльно настроенная интеллигенция", that is, a group of highly educated persons with radical progressive views and highest demands concerning the quality of the education given to their children.

[3] Vogeli mentions that "the 1921 syllabus emphasized the value of creative activity in teaching of mathematics, the need to broaden pupils' mathematical background, and the desirability of relating mathematics to life", Vogeli (1968, p. 4).

Olympiad for schoolchildren organized in biggest the Soviet cities: Leningrad, Tbilisi, Moscow in 1934–35[4]. The Olympiads helped to build traditions that went far beyond the officially stated goals (such as a high quality education). Instead, as their participants recall, they became actual festivals of mathematicians of all generations, schoolchildren, university students, school teachers, young high-school professors, and prominent scientists.

The Olympiad problems were not oriented on a mere application of school knowledge but required a capacity to find original ways of thinking, ability to reason logically in a non-standard situation. The Olympiad was usually followed by a lecture with analysis of typical mistakes and by individual meetings of the participants with the members of the jury. The Olympiad was not the only way to work informally with young talents but also a way to motivate school children to learn mathematics in a more systematic way by participating in "mathematics circles", attending public lectures given by outstanding mathematicians and by self-study of mathematical books.

Looking at the social context of this phenomenon, we can consider it as a personal mission of mathematicians contributing to the society in order to promote and popularize mathematics and emphasize the value of creative mathematical work, to search and to support young talents and give them the best of their knowledge. A mathematical community was created beyond regular educational system, and the explicitly stated goal of this community was to maintain the highest level of mathematics and to promote attractiveness of mathematical activity among the population and to support and encourage everyone with talent and interest in mathematics.

In their enrichment philosophy, Diezmann & Watters (2000) stress the need of creating opportunity for gifted children to become creative individuals. For these authors, to be creative, the individual needs an intellectual autonomy, an expertise, and a culture supportive of unconventional thought. In their study, Diezmann & Watters (2000) experimented with a special extra school science program in order to maximise the growth of creativity in gifted children basing on the development of the autonomy and domain based expertise in a social context of recognition and support. The autonomy enabled individuals to deal with novelty and generate creative products in both, evolutionary and revolutionary types of the progress. In the same order of ideas, the authors stress the importance of good thinking dependent of the problem solving context which requires either a strict application of heuristics, uncritical acceptance of information along with a disregard for contradictory evidence or it promotes non sequential process with cycles of interpretation, intuitions, and testing ideas which are typical characteristics of ill-structured problems. Finally, development of creativity relies on a social context in which gifted individuals get recognition from family and

[4] The *Encyclopedia of Young Mathematician* (*Энциклопедический словарь юного математика*, Moscow: Pedagogika, 1985, in Russian, p. 187) mentions the famous mathematicians B. N. Delone and P. S. Alexandrov as the intiators and organizers of these competitions.

teachers of their abilities and care of their development. In addition, Diezmann & Watters (2000) put emphasis on a collaborative and socially interactive learning environment as a necessary social context.

5. TOWARDS A MORE INCLUSIVE SCHOOL SYSTEM

5.1 Bringing creativity in teachers didactic inventory for gifted children

So far, we could see that in several educational systems opportunity for gifted and creative individuals were created outside or beyond the regular systems. In this section, we will analyze several in-classroom options that should be used by teachers to promote the development and nurturing of creativity in a more inclusive way.

Cline (1999) stresses the need of translating research on the creative individual, creative processes and contexts that promote creative behaviours into classroom practices providing students with opportunities to develop and demonstrate their creative abilities. According to Yastrebov (2005), the inductive nature of mathematical creativity is not being taken into account by teachers. The good understanding of dualistic nature of mathematics needs to be developed in young learners. Each mathematical fact is created by individuals. The existence of mathematics is impossible outside of specific social institution called scientific community. The scientific community approves every mathematical invention. The newly discovered mathematical fact has to be exchanged within the community being critical examined by experts in the related domain.

Guerra, Gimenez, & Servat (2005) point at familiarity, divergence and reinvention as necessary components for teacher's pedagogical knowledge. More precisely, familiarity means proposing potentially creative tasks through identification of unconventional proposals, recognizing variety of models and meanings in different contexts, openness to a variety of answers and surprising results. The divergence component enhances open debates and divergent questioning in different contexts and situations. The reinvention strategy allows choosing adequate didactic sequences to develop invention from the contexts, thinking of imaginative, real and innovative tasks, rediscovering previously learned mathematical knowledge in a new way.

Such role of teachers as promoters of creative mathematical work is crucial for developing of gifted students. According to Karp (2007, this issue), special attention has to be brought to prospective teachers' education which should enhance their didactic knowledge with examples of the ways in which mathematically gifted students construct their knowledge and use it for their further creative activity.

Many authors point at the necessity of creating more challenging mathematical classroom environments which would be suitable for all students including the gifted ones.

5.2 Creativity and thinking as components of nurturing learning-teaching environment for gifted children

Cline (1999) points at four thinking abilities to be developed which are associated with creativity, namely divergent thinking; fluency; flexibility; originality and elaboration which are fundamental elements of Guilford's (1967) definition of divergent thinking as the generation of information from given information putting emphasis on variety and quantity of output from the same source also involving transfer . In the following paragraphs, we will relate these definition to the mathematics educators' perspective on mathematical thinking.

Many authors point at the **ability to recognise patterns and to see relationships** as a *key element* in mathematical thinking. Fisher argues (1990) that since mathematics is a highly structured network of ideas, to think mathematically is to ***form connections*** in this network: the task of a teacher then is to help children to see the structure inherent in mathematics, not just rules and facts learned in isolation. He states that in encouraging children to think mathematically we need to engage all aspects of a child's intelligence. There are different ways of processing mathematics according to the following scheme (Fig.2):

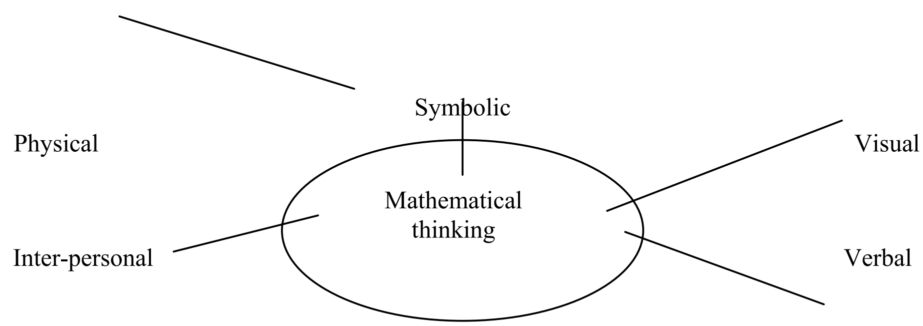

Fig.2 Different ways of mathematical thinking

We see that, according to this model, mathematical problems can be modelled or represented in a variety of ways:

- *Verbally:* through inner speech and talking things through, using linguistic intelligence, putting planning procedures and process into words, making sense and meaning for oneself
- *Inter-personally:* learning through collaboration observing others, working together to achieve a shared goal, exchanging and comparing ideas, asking questions, discussing problems
- *Physically:* using physical objects in performing mathematical tasks, working with practical apparatus, equipment and mathematical tools, modelling a problem or process, having hands-on experience, using bodily-kinesthetic skills, practical applications into the physical world

- *Visually:* putting processes into pictorial form, making drawings or diagrams visualising patterns and shapes in the mind's eye, thinking in spatial terms, graphical communication, geometric designs, manipulating mental images
- *Symbolically:* using written words and abstract symbols to interpret, record and work on mathematical problems, using different recording systems, logically exact languages, translating into mathematical codes

According to Baroody (1987), genuine learning also involves the ability in **changing of thinking patterns**. In fact, organized differently, insights can provide fresh and more powerful perspectives thus changing *how* a child thinks about something.. More specifically, making a connection can change the way knowledge is organised. The child who does not know the basic subtraction combinations uses fingers to calculate differences. For example facing the following series of subtractios: 2 - 1 =_, 4 - 2 =_, 6 - 3 =_, 8 - 4 =_, 10 - 5 = _, the child laboriously calculates each answer. Suddenly the child may have an insight: These subtraction combinations are the inverses of well-known additions of doubles (1 + 1 = 2, 2 +2 = 4, 3 + 3 = 6, 4+ 4 = 8, and 5 + 5 = 10). Thus she would produce a new relationship between subtraction combinations and the familiar addition facts which allows her to see subtraction in a different light. Given a problem like 5 - 3 = _, the child now thinks to herself; "Three plus what makes five? Oh, yeah, two." Her new perspective now enables her to solve subtraction combinations efficiently without laborious calculation. Mathematical development, then, entails qualitative changes in thinking as well as quantitative changes in the amount of information stored. Essential to the development of understanding are changes in thinking patterns. (Baroody, 1987,p.11)

According to Schrag (1988), mental activity is *purposeful* thinking only if it is experienced as *directed* to a problem or task one has set oneself. This, admittedly normative, conception is meant to include cases in which we may suddenly see a solution without any awareness of "wrestling" with a problem. But even in such cases, an idea does not appear as a *solution* unless it is experienced in relation to some difficulty one has been worrying about. Thus thinking is evoked in **situations** where one is not quite sure how to go on. Schrag calls such situations *problems*.

Referring to works of Polya (1957) and Schoenfeld (1979), Ernst (1998) points at two types of thinking activities that might affect problem-solving behaviour: cognitive and metacognitive. **Cognitive** activities include using and applying facts, skills, concepts and all forms of mathematical knowledge. They also include applying general and topic specific mathematical strategies, and carrying out problem-solving plans. **Meta-cognitive** activities, involve planning, monitoring progress, making effort calculations (e.g. 'Is this approach too hard or too slow?'), decision making, checking work, choosing strategies, and son on. Metacognition (literally: "above cognition") is about the management of thinking. Sierpinska et.al.,(2002) characterize mathematical thinking as "a good balance between **theoretical** and **practical** thinking". In their study of relationships between theoretical thinking and high achievement in linear algebra, Sierpinska et al. (2002) assume that theoretical thinking "is not a continuation but a reversal of the practical thought." (p.11). They view practical thinking as an "epistemological obstacle" that cannot and should not be avoided. However, they claim that teaching abstract mathematical concepts that puts too much emphasis on the "concrete" experience based on so-called "geometric" or "numerical" approaches might leave students with representations irrelevant from the point of view of the concepts and lead them to contradictions. (Ibid.: p.19).

Defining theoretical thinking as reflective, systemic (definitional, proof-based and hypothetical) and analytic (linguistic sensitive and meta-linguistic sensitive), these authors argue for the necessity of theoretical thinking in understanding linear algebra as following:

- The undergraduate learner of linear algebra must be even more theoretically inclined than the inventors of the theory
- Meanings of concepts must be sought in their relations with other concepts
- The learner must engage in proving activity and therefore use systemic approaches to meanings and validity
- The learner has to accept that his or her ontological questions will remain unanswered
- The learner must engage in hypothetical thinking
- The learner must become mathematically "multilingual" (Ibid.:33-35)

If we project these ideas on the elementary school level, we could see that today's tendencies (discussed in previous sections) do not favor the education of a "theoretical thinker" although our practice shows that mathematically gifted students, even at an early age hold certain epistemological views about mathematics that are close to the theoretical thinking.

When we think about the interpretation of these aspects of thinking in terms of giftedness, then we may suppose that a mathematically gifted child who is a high achiever would probably demonstrate a balanced ability to think mathematically (theoretically and practically). A mathematically able child who is not a high achiever would be rather more "theoretically inclined". The question is whether a mathematically able child can be only "practical"? Another question is to what point we can identify a child as a theoretical thinker?

Another question arises: what kind of classroom situations would enhance the fostering the development of mathematical thinking in young children?

In order to *foster* the development of mathematical thinking, Baroody (1993) stresses a use of a problem-solving approach which focuses on the processes of mathematical inquiry: problem solving, reasoning, and communicating. It is a teacher-guided approach in which a student plays an active role.

Ernst (1998) makes a comparative analysis of different teaching approaches related to the mathematical thinking. It shows the didactical transition of mathematical process which "progresses from the application of facts, skills and concepts, to a limited repertoire of problem-solving strategies including generalisation and the induction of pattern, to the full range of problem-solving strategies, and finally adding problem-posing processes as well"(p.132) happens when the classroom teaching becomes more open and challenging:

Table. 1 Ernst's comparative analysis of different teaching approaches

Teaching approach	*Role of the teacher*	*Role of the learner*	*Process involved*
Direct instruction	To state an item of knowledge explicitly. To provide exercises for application	To apply the given knowledge to exercises	The application of facts, skills and concepts
Guided discovery	To present a rule or other form of mathematical knowledge implicitly in a sequence of examples	To infer the rule or knowledge implicit in the given examples	Generalisation. Induction from pattern
Problem-solving	To present a problem to the student, leaving the solution method open	To attempt to solve the problem using own method(s)	Problem-solving strategies
Investigatory mathematics	To present an initial area of investigation, or to vet a student's own choice	To choose questions for investigation within the topic given. To explore the topic freely	Problem-posing and problem-solving strategies

Fischbein (1990) defines a teacher's task as to "create an environment that would require a mathematical attitude, mathematical concepts, and mathematical solutions". He argues that when facing a challenging task children might not be able to find solutions spontaneously. They might get engaged in a constructional process combining various conditions. They then have to produce a method to work on the problem systematically. Fischbein sees this aspect of finding a method, an algorithm used consciously as fundamental for the development of mathematical reasoning. According to Fischbein (1990), the question is whether the teacher should wait until children find the method by themselves without any help. In his opinion

> Formal reasoning doesn't develop spontaneously as a main way of thinking. This conclusion doesn't imply that the teacher should simply offer the solution. What the teacher should do is to direct the student's efforts to a solution by asking adequate questions. The student builds the answers as a reaction to a certain environment. This environment should be programmed as a problematic one in order to inspire student's solution endeavors. (Fischbein, 1990,p.8)

These theoretical guidelines cohere with Driscoll's (1999) remarks that through

- consistent modeling of algebraic thinking
- giving well timed pointers to students that help them shift or expand their thinking, or that help them to pay attention to what is important.
- making it a habit to ask a variety of questions aimed at helping students organize their thinking and respond to algebraic prompts.

Teachers would promote those habits of mind that are specific to the algebraic thinking and should be developed in children as follows:

- Reversibility as a capacity not only to use a process to get to the goal, but also to understand the process well enough to *work backward* from the answer to the starting point.

- Building rules as a capacity to *recognize patterns* and *organize data*.

- *Abstracting from calculations* as a capacity to think about computations independently of particular numbers that are used. (Driscoll, 1999,p.3)

6. CONCLUDING REMARKS

Drawing from the aforementioned theoretical considerations, we now move towards more practically oriented questions such as what are mathematical activities that would help to foster the emergence of a creative thinking in mathematically gifted elementary school students allowing them to progress in the mixed-ability classroom? Numerous studies point at mathematically rich tasks as an engine of such fostering. Peressini & Knuth (2000) mention that mathematically rich tasks are those that fit following criteria:

- encourage a range of solution approaches,
- address significant mathematical concepts,
- require students to justify their explanations,
- are open ended

The use of such tasks requires a rethinking of the role of both the teacher and the student. Burton (1984) stresses that the teacher's role shifts from that of providing information, to question-asker and resource-provider. The teacher would challenge pupils to justify or falsify arguments and to reflect on what has been done. The tone of the teacher's interventions is also important. It has to emphasise enquiry rather than instruction. Sriraman’s (2004)discussion of mathematical creativity indicates that many of the characteristics of mathematical creativity described by mathematicians as invaluable aspects of their craft such as the freedom to choose and pursue problems in an academic setting, the freedom of movement required during work, the awareness of the distinction between learning versus creating, the aesthetic appeal of mathematics and the affective urgency/drive to solve problems with tremendous real world implications, might be difficult to simulate in a traditional classroom setting. However there are basic principles which are implementable by classroom teachers. Five overarching principles that emerged from Sriraman's (2005) synthesis and analysis of the literature as significantly enhancing mathematical creativity are labeled as (1) The Gestalt principle, (2) The Aesthetic principle, (3) The Free Market principle, (4) The Scholarly principle and (5) The Uncertainty principle.

As for the student, we underline that along with sustaining interest, motivation and success in problem-solving, the following aspects are also to be mentioned:

- Choosing and using representations that would enhance the ability to model a problem
- Resolving instead of solving which helps to develop an attitude to build a network of new questions, new resolutions, and further questions out of an initial problem following a spiral development instead of a linear one (problem - solution)
- Communicating using different tools of communication

Our study of challenging situation approach within an inclusive mathematics classroom (Freiman, 2006) demonstrates that in such situations, gifted children can go further, go beyond situations, ask new questions, initiate their own investigations, and be more creative in their mathematical work. At the same time, such classroom environments turn to be nurturing for all students helping them to become more creative learners.

Many researchers in mathematics education stress an importance of specific didactical environments to nurture creative thinking in young learners. Meissner (2005) concentrates his study on three aspects:

- *individual and social components*, like motivation, curiosity, self-confidence, flexibility, engagement, humor, imagination, happiness, acceptance of oneself and others, satisfaction, success, ...
- profound discussions as well as intuitive or spontaneous *"challenging problems"* that are fascinating, interesting, exciting, thrilling, important, provoking, ...
- the students must be able to identify themselves with the problem and its possible solution(s), developing i*mportant abilities* to explore and to structure a problem, to invent own or to modify given techniques, to listen and argue, to define goals, to cooperate in teams, ...

Taking these aspects into consideration would help children to become active, to discover and to experiment, to enjoy and to have fun, to guess and to test, to laugh at their own mistakes. The recent ICMI study 16 on challenging mathematics led by P.Taylor & E. Barbeau, http://www.amt.canberra.edu.au/icmis16dd.html accentuates the need for further research on providing students with rich and challenging mathematical problem solving experiences in order to develop and nurture their creativity

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