Section 2.3 - Axioms of Probability

STAT 341: Intro Prob and Stat

The University of Montana
1. An Opening Question
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An Opening Question

Suppose that we were interested in determining the probability of rolling a product that is even when two fair six-sided dice are rolled. How could we determine the probability of this event from:

1. A frequentist approach?
2. A classicalist approach?
The classical approach to probability assumes that for each event, \( E \), in a sample space \( S \), there is a value \( P(E) \), referred to as the probability of the event \( E \). This probability function \( P \) must satisfy three axioms.
Axiom 1

\[ 0 \leq P(E) \leq 1 \]

for any event \( E \)
Axioms of (Classical) Probability

Axiom 2

\[ P(S) = 1 \]
Axiom 3

For any infinite sequence of pairwise mutually exclusive events $E_1, E_2, \ldots$ (that is, events for which $E_i E_j = \emptyset$ when $i \neq j$) we have the following:

$$P \left( \bigcup_{i=1}^{\infty} E_i \right) = \sum_{i=1}^{\infty} P(E_i)$$
Propsition 3.1

\[ P(\emptyset) = 0 \]
Proposition 3.2

For any finite sequence of \( n \) pairwise mutually exclusive events \( E_1, E_2, \ldots, E_n \) we have the following:

\[
P \left( \bigcup_{i=1}^{n} E_i \right) = \sum_{i=1}^{n} P(E_i)
\]
Suppose that a die is rolled 4 times. Let $i$ denote the number of times that the number six appears. Construct a probability function for $i = 0, 1, 2, 3, 4$.
Suppose that a study of HIV/TB coinfection in a certain population has determined the following probabilities:

\[ P(TB) = 0.8 \]
\[ P(HIV) = 0.6 \]
\[ P(TB \text{ and } HIV) = 0.3 \]

is the study legitimate?