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Motivation

Finding the largest and smallest values of a quantity is of practical importance in many contexts. Consider:

1. Medicine: the maximum dosage of a drug
2. Engineering: the minimum size of an I-beam to support a load
3. Wildlife Biology: the minimum cow/calf ratio that self sustains a herd of elk
4. Urban Planning: the traffic light configuration that maximizes traffic flow

These sorts of problems all belong to the field of mathematics that is called **optimization**
Global Maximum/Minimum

We say that $f$ has a **global maximum** at $p$ if $f(p)$ is greater than or equal to all values of $f$. 


We say that $f$ has a **global minimum** at $p$ if $f(p)$ is less than or equal to all values of $f$. 
If $f$ is continuous on the closed interval $a \leq x \leq b$ then $f$ has a global maximum and a global minimum on that interval.
Finding Global Extrema on a Closed Interval

If $f$ is continuous on the closed interval $a \leq x \leq b$ then, to find the global maximum or minimum,

1. Find the critical points of $f$ in the interval
2. Evaluate the function at the critical points and at the endpoints, $a$ and $b$

The global maximum and minimum are the maximum and minimum values of the function among those considered. A global maximum and minimum will always exist.
Finding Global Extrema on an Open Interval

If \( f \) is continuous on the open interval \( a < x < b \) (or \( -\infty < x < \infty \)) then, to find the global maximum or minimum,

1. Find the critical points of \( f \) in the interval
2. Evaluate the function at the critical points
3. Determine the behavior of \( f \) as \( x \) approaches the endpoints of the open interval \( a \) and \( b \) (or \( -\infty \) and \( \infty \))

The global maximum and minimum are the maximum and minimum values of the function among those considered...if they exist. A global maximum and minimum will not always exist.
For the function below, estimate the coordinates of all local and global maxima and minima on the interval \([0, 10]\).
Examples

Find the global maximum and minimum value of

\[ f(x) = x + \frac{3}{x} \]

on the interval \([1, 4]\)
Examples

For a positive constant $b$, the surge function $f(t) = te^{-bt}$ gives the quantity of drug in the body at a time $t \geq 0$.

1. Find the global maximum and minimum value of $f$ for $t \geq 0$.
2. Find the value of $b$ that makes $t = 10$ the global maximum.
Examples

Sketch a continuous, differentiable graph with the following properties:

- local minima at \( x = 2 \) and \( x = 4 \)
- global minimum at \( x = 2 \)
- local and global maximum at \( x = 3 \)
- no other extrema