

Problems marked by * are required for graduate students, optional for undergraduates.

From DeGroot and Schervish: Sec. 3.10 (pp. 176-9): #22; Sec. 4.1 (pp. 188-9): #3,6,8,10*; Sec. 4.2 (pp. 196-7): #3,4,6,8; Sec. 4.3 (p. 203): #3,4,6,

Additional Problem:

1. *Suppose we want to pick random points in a circular field (for example, we want to gather data from small plots centered at random points in an area so we can estimate quantities like average cover, amount of biomass, number of rare plants, etc.) One procedure would be as follows: stand in the center of the circle. Choose a random number from 0 to 2π and a second random number (independently of the first) from 0 to T where T is the radius of the circle. The first number gives the direction you walk, in radians clockwise from North. The second number gives you the distance you walk in that direction to get to the randomly selected point. Does this procedure give you a random point in the field in the sense that the probability that the point will land in any subset of the circle is proportional to the area of the subset? That is, if X and Y represent the Cartesian coordinates of a random point, is the joint distribution of X and Y uniform over the circle?
 - (a) Convince yourself and the reader that the answer to this question is “no” without doing any fancy derivations.
 - (b) Let Θ be the direction you walk and let R be the distance you walk. Then Θ is a uniform random variable over $(0, 2\pi)$ and R is a uniform random variable over $(0, T)$. For simplicity, and without loss of generality, assume $T = 1$ (R can then be thought of as the proportion of the distance to the edge you walk). Let X and Y be the Cartesian coordinates of the point. Determine the joint p.d.f. of X and Y . Verify that it is not uniform over the unit circle.
 - (c) Is there a way to modify this procedure (choosing a direction, then choosing a distance) so that the resulting distribution of (X, Y) is uniform over the unit circle?

Extra Credit

Suppose X and Y are i.i.d. $U(0, 1)$ random variables. What is the p.d.f. of $Z = X/(X - Y)$? Verify your result via simulation. (Hint: do the simulation first, to see what the p.d.f. is going to look like.)