STAT 421

Fall 2010 Homework 2, due Fri., Sept. 17

Problems marked by * are required for graduate students, optional for undergraduates.
From DeGroot and Schervish:
Sec. 1.7 (p. 27): #6,8; Sec. 1.8 (p. 34): #8,10,16,20*; Sec. 1.9 (p. 38): #2,10

Additional Problems:

1. On p. 26 of the text, a solution is given to the birthday problem.

   (a) Verify a few of the values in Table 1.1 with a simulation in R.

   (b) Estimate by simulation the probability that in a group of n people, $n = 20, 50, 100$, there will be at least three people who share the same birthday. Estimate the smallest value of n such that this probability is at least 0.5.

   (c) Can you derive an exact expression for the probability that at least 3 people in a group of n people share the same birthday? (Verify that it agrees with your simulation results.)

2. Penny Kukuk, a biologist at UM, performed the following experiment. Four adult female bees of an Australian species that builds its hives underground (Lasioglossum hemichalceum) were placed in a circular container with a smooth uniform surface of dirt with four indentations placed at the edge of the circle 90 degrees apart. The bees instinctively would burrow into the indentations (I'll call the indentations "holes" for short). Sometimes, more than one bee would burrow into the same hole leaving some holes empty. After all the bees had burrowed into the holes, the pattern of bee occupancy was observed. This experiment was repeated 37 times with different bees each time.

Pattern	Count
4,0,0,0	4
3,1,0,0	4
2,2,0,0	5
2,1,1,0	17
1,1,1,1	7

   The biologist was interested in whether the bees appear to choose holes independently of each other (the alternative is that they tend to either choose holes in which there already is a bee or tend to avoid holes in which there already is a bee). Therefore, we want to calculate the probability of each pattern if each bee chose a hole randomly and independently of the other bees.

   Note that, as far as the biologist is concerned, the holes are indistinguishable and the bees are indistinguishable. For instance, the pattern "3,1,0,0" means that 3 bees ended up in one of the holes and 1 in another; it doesn't matter which holes or which bees. However, to calculate the probabilities it is easiest to think of the bees and holes as being distinguishable.

(a) How many possible occupancy patterns are there if the bees and holes are distinguishable (e.g., bees 1 and 3 in hole A, bee 2 in hole C, bee 4 in hole B, hole D empty is one pattern)?

(b) How many of the occupancy patterns in a) lead to each of the patterns in the table above? Use combinatorial counting rather than simply listing all the possibilities (although you can check your answers that way if you wish).

(c) If the bees are selecting holes randomly and independently of each other, then all of the occupancy patterns in a) are equally likely. What are the probabilities of each of the patterns if this is the case?

(d) Compare the probabilities you calculated in c) to the observed proportions of each pattern in the 37 experiments. Does the observed behavior of the bees seem consistent with the model of random and independent selection of holes?