Counting and Probability Problems

1. How many ways can 6 kids line up at a drinking fountain?

2. How many different ways could a committee of 4 students be chosen from a class of 12 students?

3. How many different ways could a president, vice-president, treasurer, and secretary be chosen from a class of 12 students?

4. What is the probability that the first three playing cards drawn without replacement from a standard deck of cards will be all two's? Express your answer as a common fraction.

5. Six people are going to be seated in a row of 8 chairs. How many different ways can they be seated?

6. How many different ways can six people be seated in a row of seven seats if Jim and John must be in adjacent seats?

7. If six people are seated randomly in a row of seven seats, what is the probability that Jim and John end up in adjacent seats?

8. A garage door opener has a ten-digit keypad. Codes to the door must consist of 5 digits with no adjacent digits the same. How many codes are possible?

9. If six coins are tossed, what is the probability that the outcome will show 3 heads and 3 tails? Express your answer as a common fraction.

10. In a classroom, 9 students are talking, 5 are standing, and 4 are reading. One student is standing and not talking. One is reading and not talking. What is the smallest possible number of students in the room?

11. From a group of six students living in the same neighborhood, a social committee is to be appointed. Given that the committee must have at least three members, how many different committees can be formed?

12. How many distinct ways can the letters of the word PEOPLE be arranged so that the two P's are together and the two E's are together?

13. If letters of the word PEOPLE are arranged randomly, what is the probability that the two P's are together and the two E's are together?

14. I randomly pick two integers from 1 to 4 with replacement. What is the probability that the first number will be greater than the second?

15. I randomly pick two integers from 1 to 4 *without* replacement. What is the probability that the first number will be greater than the second?

16. I confiscate cell phones from four people and then randomly distribute the phones back to the people. What is the probability that at least one person receives his or her own phone?

17. A bag contains six white marbles, five red marbles, and four blue marbles. Norka will reach into the bag and pull out three marbles at the same time. What is the probability that all three marbles will be the same color? Express your answer as a common fraction.

Counting and Probability Solutions

1. Order matters so we're counting permutations and the answer is $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \mathbf{720}$

2. Order doesn't matter (Alyssa, Bob, Cathy and Doug is the same as Cathy, Bob, Doug, and Alyssa) so we want combinations: $\binom{12}{4} = \frac{12!}{4!8!} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2} = 11 \cdot 5 \cdot 9 = \mathbf{495}$.

3. Order *does* matter now: choosing Alyssa, Bob, Cathy and Doug is different than choosing Cathy, Bob, Doug, and Alyssa because they'll have different jobs. Using the Multiplication Principle, there are $12 \cdot 11 \cdot 10 \cdot 9 =$ **11,880** different ways to fill these positions.

4. $\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} = \frac{1}{13} \cdot \frac{1}{17} \cdot \frac{1}{25} = \frac{1}{5525}$

5. Treat the two empty chairs as two more people who are not distinguishable. If they were distinguishable, there would be 8! arrangements. We need to divide this by 2 since the two blank spaces can be interchanged without changing the arrangement. So 8!/2=**20,160**.

6. Consider the empty seat as a person and the pair Jim-John as one person. Then there are six "people" and so there are $6! = 720$ arrangements. However, Jim and John can switch positions in any of these arrangements so the total is $2 \cdot 720 = \mathbf{1440}$.

7. $6!/7! = 1/7$

8. There are 10 choices for the first digit, 9 for the second (all except the same one as the first), 9 for the third (all except the same one as the second), etc. So the total is $10 \cdot 9^4 =$ **65,610**.

9. There are $\binom{6}{3} = \frac{6!}{3!3!} = 20$ sequences of 3 heads and 3 tails. Each has probability $(1/2)^6$. So the probability is $20(1/2)^6 = 20/64 = \mathbf{5/16}$.

10. The key here is to recognize that the student who is standing and not talking and the student who is reading and not talking can be the same student. Then it becomes obvious that you could have just **10** students: the 9 talking and the one not talking. The readers and standers can also be talkers. You could also use a Venn diagram to allocate the students.

11. We must compute the number of committees of size 3, size 4, size 5 and size 6. There are therefore a total of $\binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6} = 20 + 15 + 6 + 1 = \mathbf{42}$.

12. Treat the two P's as one "letter" and the two E's as one "letter". Then there are four "letters" total and 4!=**24** distinct arrangements.

13. There are $6!/2!2! = 180$ equally likely distinguishable arrangements of the letters so, using the result of the previous problem, the probability is $24/180 = \mathbf{6/45}$.

14. The sample space is $\Omega = \{11, 12, 13, 14, 21, 22, 23, 24, 31, 32, 33, 34, 41, 42, 43, 44\}$ with $4^2 = 16$ equally likely outcomes. The event that the first number is greater than the second consists of the six outcomes $\{21, 31, 32, 41, 42, 43\}$. So the probability the first number will be greater than the second is $6/16 = 3/8$.

15. The sample space is $\Omega = \{12, 13, 14, 21, 23, 24, 31, 32, 34, 41, 42, 43\}$ with $4 \cdot 3 = 12$ equally likely outcomes. The event that the first number is greater than the second consists of the six outcomes $\{21, 31, 32, 41, 42, 43\}$. So the probability the first number will be greater than the second is $6/12 = 1/2$.

16. We can describe an outcome by listing in order the phone that each of the people got. For example, 2314 means that person 1 got phone 2, person 2 got phone 3, person 3 got phone 1, and person 4 got phone 4. The sample space then consists of all $4! = 24$ permutations of 1,2,3 and 4:

    $\Omega = \{1234, 1243, 1324, 1342, 1423, 1432, 2134, 2143, 2314, 2341, 2413,$
    $2431, 3124, 3142, 3214, 3241, 3412, 3421, 4123, 4132, 4213, 4231, 4312, 4321\}.$

    The probability that at least one person receives his or her own phone is $(8 + 6 + 1)/24 = 15/24 = 5/8$.

17. We can view this as a sequence of three draws without replacement. The probability of three whites is $(6/15)(5/14)(4/13)$. The probability of three reds is $(5/15)(4/14)(3/13)$. The probability of three blues is $(4/15)(3/14)(2/13)$. Therefore, the probability of three of one color is $\frac{6\cdot5\cdot4+5\cdot4\cdot3+4\cdot3\cdot2}{15\cdot14\cdot13} = \frac{6(20+10+4)}{15\cdot14\cdot13} = \frac{34}{5\cdot7\cdot13} = \mathbf{34/455}$.