1. Sally selects 2008 points in the plane so that no three are collinear. Cathy colors them so that half of the points are colored Blue while the other half are colored Green. Penelope then pairs each Blue point with a Green point and joins each pair with a line segment. Show that Penelope can do her pairing so that no two line segments intersect.

**Hint:** The 2008 points are fixed and their coloring is also fixed. Penelope may be able to eliminate an intersection by revising two pairings (see lefthand and middle figures). However, there is a possibility that a revision may create a new intersection (see righthand figure).

2. Note: $\emptyset, \{1\}, \{2\}, \{1, 2\}$ is a complete list of the subsets of $\{1, 2\}$.
   a) How many subsets of $\{M, A, T, H\}$ do not contain the letter $M$?
   b) How many subsets of $\{M, A, T, H\}$ contain at least one of $M$ or $T$?
   c) How many subsets of $\{M, A, T, H, e, m, a, t, i, c, s\}$ contain only one of $M$ or $m$?

3. Suppose $B = \left( b_{ij} \right)$ is an $n \times n$ matrix such that $\sum_{j=1}^{n} b_{ij} = 1$ for each $i$.
   Show that the matrix $C = (B - I)^2$ is not invertible. ($I$ is the $n \times n$ identity matrix.)

4. A “fair” die has its six faces labeled from zero to five. Tom rolls this die and records value on top face as his initial score. Tom rolls the die again and adds value of top face to his previous score. Tom repeats the rolling of the die and adding to his score — stopping only when his total score first exceeds twelve. (Because one face is labeled “0”, that may take many rolls.) What is the most likely value for Tom’s terminal total?

5. Suppose $0 < d_n < 1$ for all positive integers $n$. Also suppose $\sum_{n=1}^{\infty} d_n$ converges.
   Discuss convergence of $\sum_{n=1}^{\infty} \frac{d_n}{1 - d_n}$.
   Note: prove convergence or prove divergence or give examples of each behavior.
6. \(ABCD\) is a square. Point \(P\) is on line segment \(AB\), point \(Q\) is on line segment \(BC\), and line segment \(AP\) is twice as long as line segment \(BQ\); furthermore line segments \(DP\) and \(DQ\) have the same length. Compare areas of three regions: triangle \(APD\), quadrilateral \(PBQD\), triangle \(QCD\).

Marilyn vos Savant presented this problem in her Parade magazine column on 6–April–2008.

7. Show that if \(M\) is the square of an integer and if \(M\) is larger than ten, then its decimal numeral must have at least two different digits.

8. If \(a, b, c\) are positive real numbers, then the four lines \(ax \pm by \pm c = 0\) meet at four points which are vertices of a quadrilateral. Show that quadrilateral has area \(\frac{2c^2}{a \cdot b}\).

9. Let \(f(x) = 1 - e^{-1/x}\).
   a) Find the derivative of \(g(x) = \frac{1}{f(x)} = \frac{1}{1 - e^{-1/x}}\) and identify its domain.
   b) Evaluate the (improper) integral \(\int_{-1}^{1} \frac{e^{-1/x}}{x^2 \cdot (1 - e^{-1/x})^2} \, dx\).

10. Two circles are tangent internally at point \(A\). Points \(B\) and \(C\) are on the larger circle; chords \(AB\) and \(AC\) of the larger circle meet the smaller circle at points \(D\) and \(E\) respectively. Show that line segments \(BC\) and \(DE\) are parallel.

This is problem 6 on page 199 of “Plane and Solid Geometry” by H.E. Slaught and Nels Johann Lennes; 1911.

Tradition: our last problem is truly a Lennes problem.