

Math 225 – Autumn 2007
Supplementary Exercises on Predicate Logic

1. Justify each step in the following proof of

$$\exists x (P(x) \rightarrow Q(x)) \wedge \forall x P(x) \rightarrow \exists x Q(x).$$

- (a) $\exists x (P(x) \rightarrow Q(x))$
- (b) $P(a) \rightarrow Q(a)$
- (c) $\forall x P(x)$
- (d) $P(a)$
- (e) $Q(a)$
- (f) $\exists x Q(x)$

2. Prove that the following statement involving predicates is a valid argument form:

$$\forall x P(x) \wedge \exists x Q(x) \rightarrow \exists x (P(x) \wedge Q(x)).$$

3. For each of the statements below involving predicates, either prove that it is a valid argument form, or give an interpretation (common domain for the variables and meaning of each predicate) to show that it is false.

- (a) $\exists x (R(x) \vee S(x)) \rightarrow \exists x R(x) \vee \exists x S(x)$
- (b) $\forall x P(x) \vee \exists x Q(x) \rightarrow \forall x (P(x) \vee Q(x))$

4. Using predicate logic, prove that the following argument is valid; use the predicate symbols shown.

Every computer science student works harder than somebody, and everyone who works harder than any person gets less sleep than that person. Maria is a computer science student. Therefore, Maria gets less sleep than someone. ($C(x)$, $W(x, y)$, $S(x, y)$, M)

5. Using predicate logic, prove that the following argument is valid; use the predicate symbols shown.

Every ambassador speaks only to diplomats, and some ambassador speaks to someone. Therefore, there is a diplomat. ($A(x)$, $S(x, y)$, $D(x)$)