Math 442: Mathematical Statistics **II**
Spring 2009
Solutions to the in-class assignments (03/11/09).

1. Suppose that $\mathbf{x} = (2.799, 5.857, 4.018, 7.461, 3.518, 3.560, 2.304, 4.114, 3.316, 7.430)$ is an observed random sample from a gamma distribution with unknown parameters α and β. Write R-script to find the maximum likelihood estimate numerically .

   Solution. The log–likelihood function has the form

$$\ell(\alpha, \beta|\mathbf{x}) = -n \ln \Gamma(\alpha) - n\alpha \ln \beta + (\alpha - 1) \sum_{i=1}^{n} \ln x_i - \frac{1}{\beta} \sum_{i=1}^{n} x_i.$$

   Two commands in R for numerical optimization are nlm() and optim(). As these command preforms non-linear *minimization*, to locate the maximum the negative of the likelihood function should be supplied. Keeping this in mind, a self contained R functions to find MLEs of the parameters of the gamma distribution are posted on the course web site. The outputs using $\alpha = 0.1$ and $\beta = 0.1$ as starting values are as follows .

   (a) Using nlm()

```
$minimum
[1] 18.81789
$estimate
[1] 7.0780958 0.6269619
$gradient
[1] 1.184556e-07 1.499245e-06
$hessian
          [,1]      [,2]
[1,]  1.517130  15.94866
[2,] 15.948660 179.95244
$code
[1] 1
$iterations
[1] 63
```

   (b) Using optim()

```
$par
[1] 7.0794947 0.6264549
$value
[1] 18.81790
$counts function
gradient
     165       NA
$convergence [1] 0
$message NULL
$hessian
          [,1]      [,2]
[1,]  1.516971  15.96286
[2,] 15.962855 180.61795
```

Notice that changing the starting values does not change the final answer but possibly the number of iterations needed to arrive at the maximums.

□