Math 442: Mathematical Statistics **II**
Spring 2009
Solutions to the in-class assignments (03/02/09).

1. (G) Show that as $\nu \longrightarrow \infty$, the probability density function of the t Distribution, $f_T(x|\nu)$, converges to that of the standard normal, $\phi(x) = (2\pi)^{-1/2}e^{(-1/2)x^2}$.

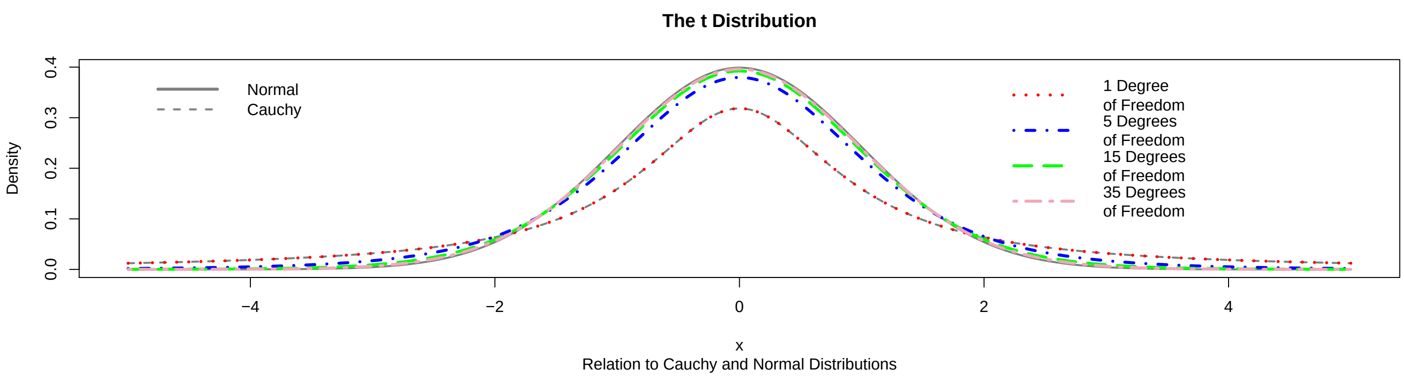
2. (G) Show that $E(|T|^k) < \infty$ for $k < \nu$ and that $E(|T|^k) = \infty$ for $k < \nu$. *Note:* this implies that moment generating function of the t distribution does not exist.

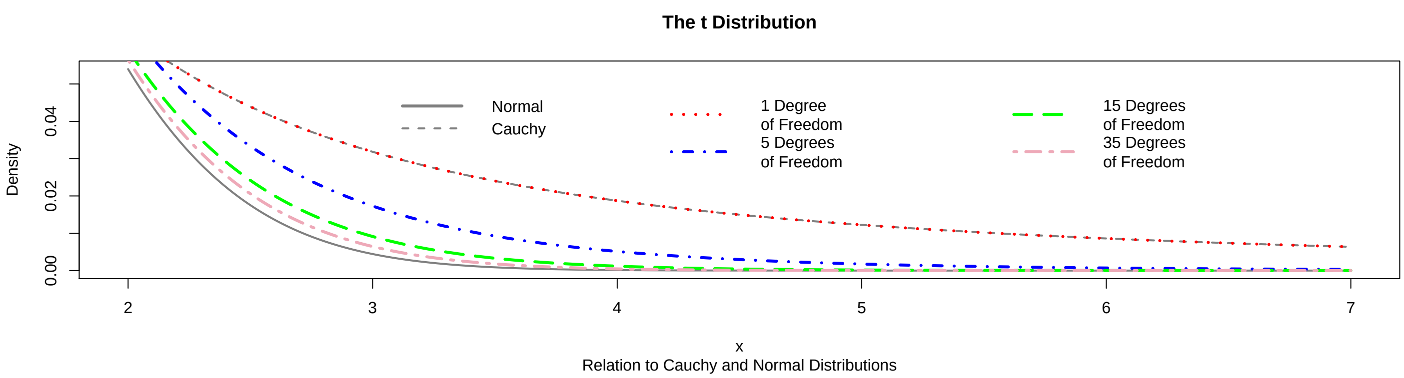
3. Use R to plot the standard normal, then overlay several t Distributions with varying degrees of freedom.

Solution. The probability density function of the t distribution is

$$f_T(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{(\nu\pi)^{1/2}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{x^2}{\nu}\right)^{-(\nu+1)/2} \qquad \text{for } -\infty < x < \infty.$$

The plot of this pdf is given below. From this plot it can be seen that $f_T(x|\nu)$ is a symmetric, bell-





shaped function with its maximum value at $x = 0$. Thus, its general shape is similar to that of the p.d.f. of a normal distribution with mean 0. However, as $x \to \infty$ or $x \to -\infty$, the tails of the p.d.f. $[f_T(x)]$ approach 0 much more slowly than do the tails of the p.d.f. of a normal distribution. In fact, it can be seen from the pdf that when $\nu = 1$, the t distribution reduces to the Cauchy distribution. Hence, when ν is large ($\nu \geq 35$ in the second plot), the t distribution with ν degrees of freedom can be approximated by the standard normal distribution. $\square$