Mathematical Statistics **II**

Spring 2009

Solutions to the in-class assignments (2/13/09).

1. Suppose that $X_1, \ldots, X_n$ form a random sample of size n from an exponential distribution with parameter β.
   Determine the distribution of the sample mean $\overline{X}_n$.

   Solution. Let $M_{X_i}(t/n)$ denote the moment generating function of X_i/n for $i = 1, \ldots, n$, and let $M_{\overline{X}_n}(t)$ denote the moment generating function of $\overline{X}_n$. Since the variables $X_1, \ldots, X_n$ are independent and identically distributed (random sample), then

$$M_{\overline{X}_n}(t) = \prod_{i=1}^{n} M_{X_i}(t/n) \ = \prod_{i=1}^{n} \frac{\beta}{\beta - t/n} \ = \left(\frac{\beta}{\beta - t/n} \right)^n .$$

   $M_{\overline{X}_n}(t)$ is the moment generating function of a gamma distribution. Hence, the sample mean $\overline{X}_n \sim$ Gamma(n, β). $\square$