Math 442: Mid Term II (Take Home)
Spring 2009.

Show all your works to receive full credit.

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with mean θ and variance σ^2. Assume that $\theta \neq 0$. Determine the asymptotic distribution of $\bar{X}_n^3$.

2. (Graduate Students Only.) Let X and Y be independent exponential random variables, with

$$f_X(x|\lambda) = \begin{cases} \frac{1}{\lambda} e^{-x/\lambda} & \text{for } x > 0 \\ 0 & \text{ow} \end{cases} \quad \text{and} \quad f_Y(y|\lambda) = \begin{cases} \frac{1}{\mu} e^{-y/\mu} & \text{for } y > 0 \\ 0 & \text{otherwise} \end{cases}.$$

   Define Z and W by

$$Z = \min\{X, Y\} \quad \text{and} \quad W = \begin{cases} 1 & \text{if } Z = X \\ 0 & \text{if} Z = Y \end{cases}.$$

   Suppose we can only observe $(Z_1, W_1), (Z_2, W_2), \ldots, (Z_n, W_n)$. Find the MLE of λ and μ from the observed data.

3. (Undergraduate Students Only.) Let X_1, X_2, ..., X_n be a sample from the *inverse Gaussian* pdf,

$$f_X(x|\mu, \lambda) = \begin{cases} \left(\frac{\lambda}{2\pi x^3}\right)^{1/2} \exp\{-\lambda(x-\mu)^2/(2\mu^2 x)\} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}.$$

   Find MLE of λ and μ.

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from

$$f_X(x|\theta) = \theta(1-\theta)^x I_{\{0,1,2,\ldots\}}(x), \quad 0 < \theta < 1$$

   where the notation $I_A(x)$ has the following definition

$$I_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \in A^c \end{cases}.$$

   Let $g(\theta) = (1-\theta)/\theta$.

   (a) Find the MOM and ML estimator of $g(\theta)$.

   (b) (Graduate students only.)Assume a Uniform$(0, 1)$ prior for θ and find the Bayes estimator of $g(\theta)$.

   (c) Is the ML estimator unbiased? Why?

   (d) Calculate the MSE of the ML estimator.

   (e) (Graduate students only.) Which of the three estimator is preferable?

5. (Computational Problem.) Suppose the proportion of survivors of melanoma in 10 randomly selected communities of Montana were 0.7013, 0.2661, 0.5751, 0.1433, 0.1929, 0.4917, 0.7922, 0.4220, 0.1841 and 0.3096.

   (a) Choose an appropriate model for this data and plot the likelihood function for the parameters of the model and comment on the plot. (attach your R-code)

   (b) Find the method of moments estimator of the model. (Give explicit analytical formula for the MOM estimators and also the numerical estimates.)

   (c) Find MLE of the parameters and the chance of survivorship for a randomly selected community is less than 50%. (attach your R-code and output with brief discussion)

   (d) Conduct a small scale simulation to see if the MLE of the parameters are unbiased and/or consistent.