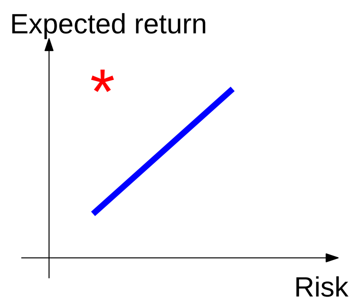
MATH 150: Answers to even numbered HW assignment problems

Section 1.1

4: (a) When $1000 is spent on advertising, the number of sales per month is 3500; (b) I; (c) the number of sales if no money is spent on advertising.

10: $f(5) = 4.1$.

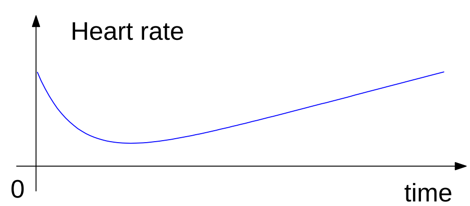
12 (a) and (b):



14: (a) At $t = 30$ minutes the temperature was 10^0C. (b) Initial temperature of the object was a^0C. At time $t = b$ the object's temperature is 0^0C.

18: (a) III; (b) Vertical intercept represents the temperature of potato at $t = 0$.

20:



Section 1.2

2: Slope is $-3/2$, vertical intercept is 4.

8: $y = -7 + 3x$.

12: (a) $P = 30,700 + 850t$; (b) $P(10) = 39,200$. (c) Population will reach $45,000$ in about 16.82 years ($t = 16.82$), or during the year 2016.

Section 1.3

2: concave up.

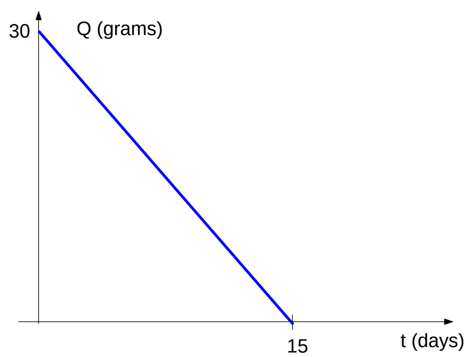
6: (a) intervals between D and E, between H and I; (b) intervals between A and B, between E and F; (c) intervals between C and D, between G and H; (d) intervals between B and C, between F and G;

18: (a) 16.4 billion \$; (b) 3.28 billion \$ per year; (c) -6.0 billion \$ between 2000 and 2001.

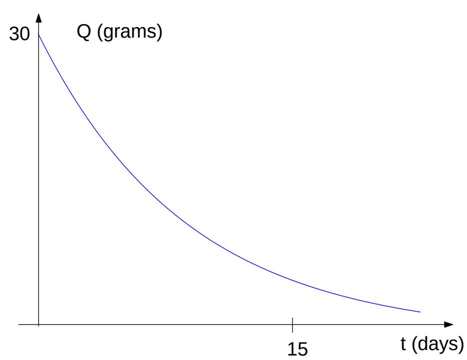
Section 1.5

2: (a) initial amount = 100; growth; growth rate = 7 % = 0.07; (b) initial amount = 5.3; growth; growth rate = 5.4 % = 0.054; (c) initial amount = 3500; decay; decay rate = - 7 % = - 0.07; (b) initial amount = 12; decay; decay rate = -12 % = 0.12.

6: (a) $Q = 30 - 2t$,



(b) $Q = 30 \times (0.88)^t$,



10: (a) II; (b) I; (a) III; (b) V; IV shows exponential decay; VI shows linear decay.

18: (a) neither; (b) exponential: $s(t) = 30.12 \times (0.6)^t$; (c) linear: $g(u) = -1.5u + 27$.

24: (a) The investment was worth \$ 3486.78 after 10 years; (b) it will take about 11 years to get the investment back to \$ 10,000.

Section 1.6

8: $t = \ln 10 \approx 2.3$.

22: (a) Town D is growing the fastest. (b) Town C is largest at $t = 0$. (c) Town B is decreasing in size.

24. $P = 2 \times (1/\sqrt{e})^t \approx 2 \times (0.6065)^t$.

Section 1.8

2: (a) $f(t+1) = (t+1)^2 + 1 = t^2 + 2t + 2$, (b) $f(t^2+1) = (t^2+1)^2 + 1 = t^4 + 2t^2 + 2$, (c) $f(2) = 5$, (d) $2f(t) = 2t^2 + 2$, (e) $[f(t)]^2 + 1 = t^4 + 2t^2 + 2$.

8: (a) $f(g(x)) = 2x^2 + 12x + 18$, (b) $g(f(x)) = 2x^2 + 3$, (c) $f(f(x)) = 8x^4$.

12: (a) $y = u^6$, where $u = 5t^2 - 2$; (b) $P = 12e^u$, where $u = -0.6t$; (c) $C = 12\ln(u)$, where $u = q^3 + 1$.

30: (a) $f(g(0)) = f(2) = 3$; (b) $f(g(1)) = f(3) = 4$; (c) $f(g(2)) = f(5) = 11$; (d) $g(f(2)) = g(3) = 8$; (e) $g(f(3)) = g(4) = 12$.

Section 1.9

2: yes, $y = 3x^{-2}$, $k = 3$, $p = -2$.

6: yes, $y = (5/2)x^{-1/2}$, $k = 5/2$, $p = -1/2$.

14: $E = kv^3$, where k is a constant.

Review for Chapter 1

6: (b) 200; (c) 80; (d) $0 \leq 0 \leq 560$; (e) decreasing; (f) concave down.

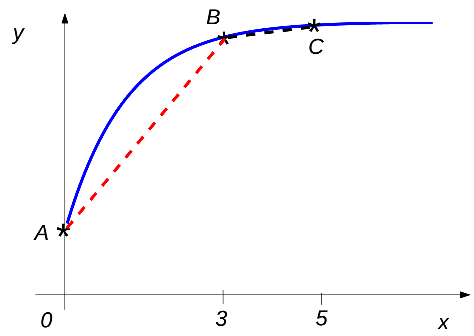
24: $y = 0.4x + 2$.

32: $g(x) = 30.8 - 3.2x$ could be linear; $h(x) = 15 \cdot 0.6^x$ could be exponential; $f(x)$ is neither.

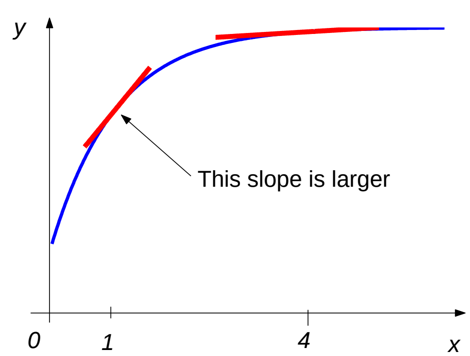
Section 2.1

4: slope -3: point F; slope -1: point C; slope 0: point E; slope 1/2: point A; slope 1: point B; slope 2: point D.

6: (a) The average rate of change between $x = 0$ and $x = 3$ is greater than the average rate of change between $x = 3$ and $x = 5$ since slope of AB > slope of BC



(b) The function is increasing faster at $x = 1$ than at $x = 4$. Thus, instantaneous rate of change at $x = 1$ is greater than that at $x = 4$.



14:

$$\begin{array}{rccccc} x: & d & b & c & a & e \\ f'(x): & 0 & 0.5 & 2 & -0.5 & -2 \end{array}$$

24: (a) $f(4) > f(3)$; (b) $f(2) - f(1) > f(3) - f(2)$; (c)

$$\frac{f(2) - f(1)}{2 - 1} > \frac{f(3) - f(1)}{3 - 1};$$

(d) $f'(1) > f'(4)$.

Section 2.2

10: IV.

14: (a) $f'(2) \approx 3$; (b) $f'(x)$ is positive for $0 < x < 4$ and is negative for $4 < x < 12$

Section 2.3

4: (a) $f(200) = 350$ means that it costs \$350 to produce 200 gallons of ice cream; (b) $f'(200) = 1.4$ means that when the number of gallons produced is 200, it costs \$1.4 to produce an additional gallon.

20: (a) $f(140) = 120$ means that a patient weighing 140 pounds should receive a dose of 120 mg; $f'(140) = 3$ tells us that if the weight of a patient increases by 1 pound (from 140 pounds), the dose should be increased by 3 mg; (b) $f(145) \approx 120 + 3 \times 5 = 150$ mg.

22: $f(22) \approx f(20) + f'(20) \cdot 2 = 345 + 6 \cdot 2 = 357$.

Section 2.4

2: $f'(x) > 0$, $f''(x) > 0$.

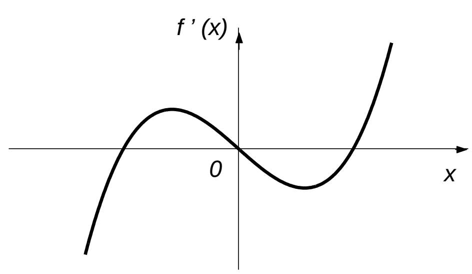
4: $f'(x) < 0$, $f''(x) = 0$.

10: $f'(t) > 0$ on the intervals $0 < t < 0.4$ and $1.7 < t < 3.4$; $f'(t) < 0$ on the intervals $0.4 < t < 1.7$ and $3.4 < t < 4$; $f''(t) > 0$ on the interval $1 < t < 2.6$; $f''(t) < 0$ on the intervals $0 < t < 1$ and $2.6 < t < 4$.

20: (a) $f(x) < 0$ at x_4 and x_5; (b) $f'(x) < 0$ at x_3 and x_4; (c) $f(x)$ is decreasing at x_3 and x_4; (d) $f'(x)$ is decreasing at x_2 and x_3; (e) slope of $f(x)$ is positive at x_1, x_2 and x_5; (f) slope of $f(x)$ is increasing at x_1, x_4 and x_5.

Review for Chapter 2

10:



18: $f(26) \approx f(25) + f'(25) \times 1 = 3.4$; $f(30) \approx f(25) + f'(25) \times 5 = 2.6$.

#20: (a) $f(1800) = 155$: consuming 1800 Calories per day results in a weight of 155 pounds; $f'(2000) = 0$: consuming 2000 Calories per day causes neither weight gain nor loss; (b) units of dW/dc are pounds/(Calories/day).

26: B.

Section: Focus on Theory (Chapter 2, pp. 135 – 140)

10: $0 \leq x \leq 2$: NO, $0 \leq x \leq 0.5$: YES.

16: YES.

24:

$$f'(x) = \lim_{h \to 0} \frac{5(x+h) - 5x}{h} = \lim_{h \to 0} 5 = 5.$$

32:

$$f'(x) = \lim_{h \to 0} \frac{2(x+h)^2 + (x+h) - 2x^2 - x}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 + h}{h} = \lim_{h \to 0} (4x + 2h + 1) = 4x + 1.$$

Section 3.1

4: $y' = -12x^{-13}$.

14: $y' = 24t^2 - 8t + 12$.

16: $y' = -12x^3 - 12x^2 - 6$.

24:

$$y' = 2z - \frac{1}{2z^2}.$$

Section 3.2

6: $f' = 5 \cdot 5^x \ln(5) + 6 \cdot 6^x \ln(6)$.

16:

$$y' = (\ln 10) \cdot 10^x - \frac{10}{x^2}.$$

18: $D' = -1/p$.

22: $f' = Ae^t + B/t$.

26: $f(0) = 1040$ megawatts; $f(15) = 1040 \cdot (1.3)^15 = 53,233$ megawatts; $f'(0) = 1040 \ln(1.3) = 273$ megawatts/year; $f'(15) = 1040 \cdot \ln(1.3) \cdot (1.3)^15 = 13,967$ megawatts/year.

Section 3.3

6: $w' = 15 \cdot (5r - 6)^2$.

16: $f' = 30E^{5x} - 2xe^{-x^2}$.
22:
$$f' = \frac{2t}{t^2+1}.$$
30: $y' = 2(5+e^x)e^x$.
38: $f(4) = 27e^{-0.14\cdot(4)} = 15.4$ ng/ml; $f'(4) = -3.78e^{-0.14\cdot(4)} = -2.16$ ng/ml per hour.

Section 3.4
8: $y' = e^t(t^2+2t+3)$.
16:
$$f' = \frac{e^{-z}}{2\sqrt{z}} - \sqrt{z}e^{-z}.$$
26:
$$y'(x) = \frac{e^x}{(1+e^x)^2}.$$
28:
$$y'(z) = \frac{z\ln z - 1 - z}{z(\ln z)^2}.$$

Section 3.5
6: $y' = 5\cos x - 5$.
8: $R' = 5\cos(5t)$.
14: $y' = 12\cos(2t) - 4\sin(4t)$.

Review for Chapter 3
8: $s' = 15t^2 - 2t + 20$.
20: $R' = 5(\sin t)^4 \cdot (\cos t)$.
36:
$$h' = \frac{2p}{(3+2p^2)^2}.$$

Section 4.1
8: $f'(x) = 3x^2 - 6$, and thus, $f'(x) = 0$ at $x = \pm\sqrt{2}$. $f'(x)$ changes from positive to negative at $x = -\sqrt{2}$, and so there is a local maximum at $x = -\sqrt{2}$. $f'(x)$ changes from negative to positive at $x = +\sqrt{2}$, and so there is a local minimum at $x = +\sqrt{2}$.

12: $f'(x) = \ln x + 1$, so $f'(x) = 0$ when $\ln x = -1$, that is, for $x = e^{-1} \approx 0.37$. This is the only place where f' changes sign. Since $f'(1) > 0$, the function f increases for $0 < x < e^{-1}$ and increases for $x > e^{-1}$. Thus, we have local minimum at $x = e^{-1}$.

Section 4.2

16: critical points at $x = 1$ (local min) and at $x = 0$ (neither min nor max); inflection points at $x = 0$ and at $x = 2/3$.

Section 4.3

22: Local and global maximum at $x = 2$.

30:

$$\frac{dE}{dF} = 0.25 - \frac{2 \cdot (1.7)}{F^3} = 0,$$

thus,

$$F = \left(\frac{2 \cdot (1.7)}{0.25}\right)^{1/3} = 2.4 \quad \text{hours.}$$

This gives a local and global minimum.

44: (a) At $t = 0$ we have $q(0) = 0$; (b) Maximum value occurs at $t = \ln 2 = 0.69$ (where $q'(t) = 0$); maximum value is $q(\ln 2) = 5$ mg; (c) as $t \to \infty$ we have $q(t) \to 0$.

Section 4.4

2: Profit function is positive for $5.5 < q < 12.5$ (when $R(q) > C(q)$), and negative for $0 < q < 5.5$, $q > 12.5$ (when $R(q) < C(q)$). Profit maximized when $R(q) > C(q)$ and $R'(q) = C'(q)$ which occurs at about $q = 9.5$.

16: Profit is maximized at $q = 75$; profit $= R(75) - C(75) = \$\ 6875$.

Section 4.8

6: (a) At peak concentration $C'(t) = 0$; corresponding $t = 1/0.03 = 33.3$ minutes; $C(33.3) \approx 245$ ng/ml; (b) after 15 min $C(15) \approx 191$ ng/ml; after one hour $C(60) \approx 198$ ng/ml.

Review for Chapter 4

8: (a) Increasing for all x. (b) No max or min.

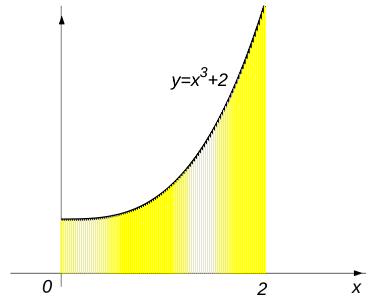
10: (a) Decreasing for $x < -1$; $0 < x < 1$; increasing for $x > 1$; $-1 < x < 0$. (b) $f(-1)$ and $f(1)$ are local minima; $f(0)$ is a local maximum.

Section 7.1
6: $t^8/8 + t^4/4 + C.$
14:
$$\frac{2}{3}z^{3/2} + C.$$
18: $\ln|z| + C.$
46:
$$\frac{e^{2t}}{2} + C.$$
50: $2x^4 + \ln|x| + C.$

Section 7.2
4: $e^{5t+2} + C.$
12: $\frac{1}{6}(x^2+3)^3 + C.$
20:
$$-\frac{1}{8}(\cos\theta + 5)^8 + C.$$
34:
$$\frac{1}{2}\ln(y^2+4) + C.$$
36: $2e^{\sqrt{y}} + C.$
38: $\ln(2 + e^x) + C.$

Section 7.3
6:
$$\int_1^4 \frac{1}{\sqrt{x}}dx = 2.$$
10:
$$\int_1^2 5t^3 dt = \frac{75}{4}.$$
22:
$$\int_0^2 x(x^2+1)^2 dx = \frac{62}{3}.$$

Section 5.3
2: Area = 8.



\# 4: Negative.
\# 6: Zero.
\# 16: (a) 13; (b) -2; (c) 11; (d) 15.

Section 5.4

\# 4: The change in velocity between times $t = 0$ and $t = 6$ hours; it is measured in km/hr.
\# 12: 2627 acres.

Section 5.5

\# 6: The fixed cost is \$500. Total variable cost is $\approx$ \$866.7. Total cost is \$500 + \$866.7 = \$1,366.7.

Review for Chapter 5

\# 10:

$$\int_1^5 (x^2+1)dx = \left(\frac{x^3}{3} + x\right)\Bigg|_1^5 \approx 45.33.$$

\# 14:

$$\int_1^3 (z+1/z)dz = \left(\frac{z^2}{2} + \ln(z)\right)\Bigg|_1^3 \approx 5.10.$$

\# 32: Since $f(x) = \sqrt{1+x^3}$ is increasing for $0 \le x \le 2$, we have

$$\int_0^2 f(0)dx \le \int_0^2 f(x)dx \le \int_0^2 f(2)dx,$$

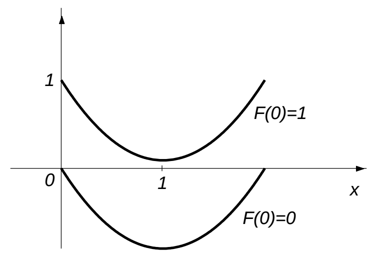
so that

$$2 = \int_0^2 dx \le \int_0^2 f(x)dx \le \int_0^2 3dx = 6.$$

\# 36: Total cost is 4,250,000 riyals.

Section 6.1
2:
$$\text{Average value } = \frac{1}{3}\int_0^3 f(x)dx = 8.$$
4: Average value $= 2$.
6: Average value ≈ 2202.55.
12(a): Average inventory ≈ 527.25.
22: (c) $<$ (a) $<$ (b).

Section 7.4
6:



16: $F(1) = -1,\ F(3) = 7,\ F(1) = -1,\ F(4) = 5.$

Review for Chapter 7
6:
$$\ln|x| - \frac{1}{x} - \frac{1}{2x^2} + C.$$

20: $3\sin(x) + 7\cos(x) + C$.
36: $\sqrt{x^2+4} + C$.

Section 10.1
2: (a) (III); (b) (V); (c) (I); (d) (II); (e) (IV).
8: Rate of change of B = Rate in − Rate out:
$$\frac{dB}{dt} = 0.04B - 2000.$$

Section 10.2
16: E.

18: A.
22: (a) II; (b) I.

Section 10.4

2: $w = 30e^{3r}$.
4: $Q = 50e^{t/5}$.
6: $p = 164.87e^{-0.1q}$.

Section 10.5

4: $P = 104e^{t} - 4$.
6: $Q = 400 - 350e^{0.3\,t}$.