Comments: HW1 John Bardsley 495

1. a) b) Note the closeness of the values when you use the nonparametric vs. the theoretical confidence interval endpts. and the mean.

c) In all cases, the null (that the residuals are normally distributed) is not rejected.

2. 95% confidence intervals & mean

Monte Carlo: (b.1) 0.75391, 0.94948, 1.2758

(b.2) 0.06716, 0.10464, 0.14442

Bootstrap: (b-1) 0.80053, 0.95346, 1.18290

(b-2) 0.07343, 0.10179, 0.13041

Normal Theory:

(b-1) 0.54901, 0.9293, 1.27568

(b-2) 0.04191, 0.10399, 0.16608

(*) Note that the widest interval comes from the normal theory; next is the monte carlo sampling; and the narrowest comes from bootstrapping. I worry that with bootstrapping & only 5 residual pts, the sample set isn't rich enough, leading to a narrower interval.

3. The same comment can be made here. Bootstrapping yields much narrower confidence intervals than the others.

See m-files on web site

4. $$r(\beta) = \begin{bmatrix} y_1 - X(\beta, t_1) \\ \vdots \\ y_{12} - X(\beta, t_{12}) \end{bmatrix} \overset{def}{=} y - X(\beta, t),$$

where $\beta = \begin{pmatrix} X_0 \\ m \\ k \end{pmatrix}$, $t = \begin{pmatrix} t_1 \\ t_2 \\ \vdots \\ t_{12} \end{pmatrix}$, and

$$X(\beta, t) = \frac{m X_0 e^{kt}}{m + X_0(e^{kt}-1)} = \frac{m X_0}{(m-1)e^{-kt} + X_0}$$

The Jacobian of $r(\beta)$ w.r.t. β is defined

$$J(\beta) = \left[-\frac{\partial X}{\partial X_0}(\beta, t), -\frac{\partial X}{\partial m}(\beta, t), -\frac{\partial X}{\partial k}(\beta, t) \right]$$

$$= \left[\frac{-m^2 e^{-kt}}{((m-X_0)e^{-kt}+X_0)^2}, \frac{-X_0^2(e^{-kt}-1)}{((m-X_0)e^{-kt}+X_0)^2}, \frac{-m X_0 (m-X_0) t e^{-kt}}{((m-X_0)e^{-kt}+X_0)^2} \right]$$

Each is a 12 element column vector

5 (a) $$\frac{dX}{dt} = \frac{d}{dt}\left[\frac{m X_0}{(m-X_0)e^{-kt}+X_0} \right] = \frac{-m X_0(-k(m-X_0)e^{-kt})}{((m-X_0)e^{-kt}+X_0)^2}$$

$$= \frac{m^2 k X_0 e^{-kt} - m k X_0^2 e^{-kt}}{((m-X_0)e^{-kt}+X_0)^2} = \boxed{\frac{m k X_0 e^{-kt}(m-X_0)}{((m-X_0)e^{-kt}+X_0)^2}}.$$

(a)

$$kX\left(1-\frac{X}{M}\right) = \frac{kMX_0}{(M-X_0)e^{-kt}+X_0}\left(1-\frac{X_0}{(M-X_0)e^{-kt}+X_0}\right)$$

$$= \boxed{\frac{kMX_0\,(M-X_0)e^{-kt}}{\left((M-X_0)e^{-kt}+X_0\right)^2}} \quad (b)$$

Since (a) = (b)

$$\frac{dX}{dt} = kX\left(1-\frac{X}{M}\right)$$

and so ~~the~~ X is a solution. Also

$$X(0) = \frac{MX_0}{(M-X_0)+X_0} = X_0$$

So the initial condition is also satisfied

(b) $$\frac{dy}{dt} = -\alpha e^{-(t/\sigma)^\gamma}\frac{d}{dt}\left[-(t/\sigma)^\gamma\right]$$

$$= \alpha\left(\gamma(t/\sigma)^{\gamma-1}\frac{1}{\sigma}\right)e^{-(t/\sigma)^\gamma}$$

$$= \boxed{\frac{\alpha\gamma}{\sigma}(t/\sigma)^{\gamma-1}e^{-(t/\sigma)^\gamma}} \quad (a)$$

$$\frac{\gamma}{\sigma^\gamma}t^{\gamma-1}(\alpha-y) = \boxed{\frac{\alpha\gamma}{\sigma}(t/\sigma)^{\gamma-1}e^{-(t/\sigma)^\gamma}} \quad (b)$$

Since (a) = (b), y is a solution. Moreover

$$y(0) = 0,$$

so the initial conditions are also satisfied.