Calculus Review #1, Differentiation

Rules for Differentiation: Let $f(t)$ and $g(t)$ be differentiable functions. Then

$$\begin{aligned}
\frac{d}{dt}[cf(t)] &= cf'(t), \quad c \text{ a constant}, \\
\frac{d}{dt}[f(t)+g(t)] &= f'(t)+g'(t), \\
\frac{d}{dt}[f(t)-g(t)] &= f'(t)-g'(t), \\
\frac{d}{dt}[f(t)g(t)] &= f'(t)g(t)+f(t)g'(t), \\
\frac{d}{dt}\left[\frac{f(t)}{g(t)}\right] &= \frac{g(t)f'(t)-f(t)g'(t)}{g(t)^2} = \frac{\text{low} \cdot (\text{d high}) - \text{high} \cdot (\text{d lo})}{\text{low}^2}. \\
\frac{d}{dt}[f(g(t))] &= f'(g(t)) \cdot g'(t) \qquad \textbf{CHAIN RULE}.
\end{aligned}$$

Some Derivatives You Should Know:

$$\begin{aligned}
\frac{d}{dt}[t^n] &= nt^{n-1}, \\
\frac{d}{dt}[e^t] &= e^t, \\
\frac{d}{dt}[\ln(t)] &= \frac{1}{t}, \\
\frac{d}{dt}[\cos(t)] &= -\sin(t), \\
\frac{d}{dt}[\sin(t)] &= \cos(t).
\end{aligned}$$

Examples (all but the first involving the chain rule):

$$\begin{aligned}
\frac{d}{dt}[7t^5-3t^4+13t^3-5t^2+t+11] &= 35t^4-12t^3+39t^2-10t+1, \\
\frac{d}{dt}[e^{2t+1}] &= e^{2t+1}\frac{d}{dt}[2t+1] = 2e^{2t+1}, \\
\frac{d}{dt}[\ln(7t-2)] &= \frac{1}{7t-2}\frac{d}{dt}[7t-2] = \frac{7}{7t-2}, \\
\frac{d}{dt}[\cos(6t^2+t)] &= -\sin(6t^2+t)\frac{d}{dt}[6t^2+t] = -(12t+1)\sin(6t^2+t), \\
\frac{d}{dt}[\sin(5t)] &= \cos(5t)\frac{d}{dt}[5t] = 5\cos(5t).
\end{aligned}$$

Calculus Review #2, Integration

Rules for Integration: Let $f(t)$ and $g(t)$ be integrable functions. Then

$$\begin{aligned}
\int [cf(t)]dt &= c\int f(t)dt, \quad c \text{ a constant},\\
\int [f(t)+g(t)]dt &= \int f(t)dt + \int g(t)dt,\\
\int [f(t)-g(t)]dt &= \int f(t)dt - \int g(t)dt,\\
\int [f(g(t))]g'(t)\,dt &= \int f(u)\,du \quad \text{where} \quad u=g(t) \qquad \textbf{u-SUBSTITUTION}
\end{aligned}$$

Some Integrals You Should Know:

$$\begin{aligned}
\int t^n\,dt &= \frac{1}{n+1}t^{n+1}+C,\\
\int e^t\,dt &= e^t+C,\\
\int \frac{1}{t}\,dt &= \ln|t|+C,\\
\int \cos(t)\,dt &= \sin(t)+C,\\
\int \sin(t)\,dt &= -\cos(t)+C.
\end{aligned}$$

Examples (the second two using u-substitution:)

$$\int (-3t^4+13t^3-5t^2+t+11)\,dt = -\frac{3}{5}t^5+\frac{13}{4}t^4-\frac{5}{3}t^3+\frac{t^2}{2}+11t+C,$$

$$\begin{aligned}
\int e^{2t+1}\,dt &= \frac{1}{2}\int 2e^{2t+1},dt\\
&= \frac{1}{2}\int e^u du \quad (u=2t+1,\ du=2dt)\\
&= \frac{1}{2}e^u+C\\
&= \frac{1}{2}e^{2t+1}+C.
\end{aligned}$$

$$\begin{aligned}
\int \frac{1}{3t+2}\,dt &= \frac{1}{3}\int \frac{1}{3t+2}3dt\\
&= \frac{1}{3}\int \frac{1}{u}du \quad (u=3t+2,\ du=3dt)\\
&= \frac{1}{3}\ln|u|+C\\
&= \frac{1}{3}\ln|3t+2|+C.
\end{aligned}$$