Interpolating Polynomials and Splines

MATH 471, NUMERICAL ANALYSIS LAB

Worksheet #9

1. The Lagrange polynomial for interpolating a function f at points $(x_1, \ldots, x_n)$ is given by:

$$p_{\mathrm{L}}(x) = \sum_{i=1}^{n} \left(\prod_{j=0,\neq i}^{n} \frac{x - x_j}{x_i - x_j} \right) f(x_i)$$

   Suppose $f(x) = e^x$. Find the first order Lagrange polynomial interpolating f at $x = 0$ and $x = e$.

2. In class we derived the first order Hermite polynomial interpolating f at $x = 0$ and $x = e$. It had the form

$$p_{\mathrm{H}}(x) = (1 + 2x)(1 - x)^2 + e(3 - 2x)x^2 + x(1 - x)^2 + ex^2(x - 1).$$

   Plot f, p_{L} and p_{H} on the interval $[0, 1]$.

3. Let's have a look at the problem on the following web site:
   `http://vortex.bd.psu.edu/~sas56/Research/index.html`

4. Interpolate the points generated using the commands

```
>> x = 0:10;
>> y = sin(x);
```

   with MATLAB's `spline` function.

5. Do the same with the points

```
>> x = -4:4;
>> y = [0 .15 1.12 2.36 2.36 1.46 .49 .06 0];
```

   And the again using `[0 y 0]` instead. Do you notice a difference?

6. And finally, try it with the points

```
>> x=-3:3;
>>y=[-1 -1 -1 0 1 1 1];
```