Computer Lab #7: MATH 471
Monday, October 29, 2008.

1. **Return to the following problem from the homework:** Modify `AOTwoD.m` so that it uses the QR factorization for solving

$$\arg\min_{\mathbf{p}} \left\| \begin{bmatrix} \mathbf{D}_x \\ \mathbf{D}_y \end{bmatrix} \mathbf{p} - \begin{bmatrix} \mathbf{g}_x \\ \mathbf{g}_y \end{bmatrix} \right\|$$

where $\mathbf{p}$ denotes the unknown phase, $\begin{bmatrix} \mathbf{g}_x \\ \mathbf{g}_y \end{bmatrix}$ the data, and $\begin{bmatrix} \mathbf{D}_x \\ \mathbf{D}_y \end{bmatrix}$ the discrete gradient operator. Hand in a listing of your code. Note that this is done automatically when you type

```
>> recon = [Dx;Dy]\data(:);
```

2. Modify `taylor3method.m` so that it implements the third-order Runge-Kutta method

$$x_{k+1} = x_k + \frac{1}{9}(2F_{1,k} + 3F_{2,k} + 4F_{3,k}),$$

where

$$\begin{aligned} F_{1,k} &= hf(t_k, x_k) \\ F_{2,k} &= hf(t_k + \frac{1}{2}h, x_k + \frac{1}{2}F_{1,k}) \\ F_{3,k} &= hf(t_k + \frac{3}{4}, x_k + \frac{3}{4}F_{2,k}) \end{aligned}$$

for solving

$$\frac{dx}{dt} = f(t,x), \quad x(t_0) = x_0.$$

Test your method on the ODE test problem from the last lab:

$$\frac{dx}{dt} = -x + \cos(t) + t^2, x(0) = 1,$$

on $[0,4]$ with $h = 0.01$. Compare your results with those of `taylor3method.m` and the analytic solution

$$x(t) = \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t) + t^2 - 2t + 2 - \frac{3}{2}e^{-t}.$$