Computer Lab #5: MATH 471
Wednesday, October 8, 2005.

Again, we consider the problem of computing wavefront errors that is a requirement of adaptive optics systems. The differential equation that must be solved, has the form

$$\nabla^2 u(x,y) = \nabla \cdot g(x,y), \quad 0 < x, y < 1, \tag{1}$$

where $\nabla^2 = \partial/\partial x^2 + \partial/\partial y^2$ is the Laplacian operator, $\nabla\cdot$ divergence operator, and $u = 0$ on the boundary of $\Omega = [0,1] \times [0,1]$. Discretization of (1) yields a linear system of the form

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \tag{2}$$

where $A \in \mathbb{R}^{N\times N}$ is the discrete version of the ∇^2 operator; $\mathbf{b} \in \mathbb{R}^N$ is the discretization of $\nabla \cdot g$, and $\mathbf{x} \in \mathbb{R}^N$ is the unknown solution.

Previously, we solved this system using Gaussian elimination of the Cholesky factorization, but this time we seek approximate solutions via iterative methods.

PROBLEM:

1. (a) Modify `AOTwoD.m` so that it uses the Jacobi iteration to approximately solve (2). The necessary syntax in MATLAB is as follows:

```
>> [x_k,error_vec]=jacobi(zeros(size(x)),A,b,10000);
```

   (b) Modify `jacobi.m` so that it implements the SOR method

$$(\mathbf{D} - \omega\mathbf{L})\mathbf{x}_k = \omega\left(\mathbf{U}\mathbf{x}_{k-1} + \mathbf{b}\right) + (1-\omega)\,\mathbf{D}\mathbf{x}_{k-1}.$$

   (Be sure to rename your new code.) Note that if $\omega = 1$, SOR is Gauss-Seidel. Use your code to solve (2) within `AOTwoD.m` as in the previous problem. What is the (approximate) optimal relaxation parameter $0 < \omega < 2$ in terms of convergence? To determine this use the `semilogy` command in MATLAB to plot the relative error for a range of ω values. The optimal parameter then corresponds to the fastest convergence.

   (c) Modify `jacobi.m` so that it implements the Richardson iteration. (Be sure, once again, to rename your new code.) Then use it within `AOTwoD.m`. What happens? Can you explain your results.