ASSIGNMENT #5: MATH 471
Due: Monday, October 20, 2008.

1. Load the data in `data.mat` on the web into MATLAB. You'll find that it contains 100,000 points in $\mathbb{R}^3$. Plot the data using the `plot3` function. Compute a PCA of the data and compress the data onto the PCA vectors corresponding to the largest two values in the PCA spectrum. Plot the compressed data in a separate figure. Hand in a listing of your code. What percentage of variation is lost in the data with the compression?

2. Modify `jacobi.m` from the web site so that it implements the SSOR method iterative method. (Be sure to rename your new code.) Note that if $\omega = 1$, SOR is Gauss-Seidel. Use your code to solve the linear system within `AOTwoD.m` as in the previous problem. What is the (approximate) optimal relaxation parameter $0 < \omega < 2$ in terms of convergence? To determine this use the `semilogy` command in MATLAB to plot the relative error for a range of ω values. The optimal parameter then corresponds to the fastest convergence.

3. (a) Apply conjugate gradient to the linear system in `AOTwoD.m`.
   (b) Apply the Q matrix from SSOR algorithm as a preconditioner for CG on this problem. What is the (approximate) optimal relaxation parameter $0 < \omega < 2$ in terms of CG convergence? Plot CG convergence for a number (say 4) choices of $0 < \omega < 2$ as well as for CG without preconditioning.
   (c) How does preconditioned CG convergence compare to SSOR convergence? Back up your claim.

4. Section 4.6, Problems 13, 14, 16, 17. These are proof problems and they should go quickly.