ASSIGNMENT #2: MATH 471
Due: Friday, September 20, 2005.

1. Let

$$f(x) = \frac{x^3 + 4x^2 + 3x + 5}{2x^3 - 9x^2 + 18x - 2}, \tag{1}$$

   (a) Using `newton.m` from the web site, find a root of f. Discuss how your initial guess for the iterations was chosen. Cut and paste the MATLAB output into a text document and hand that in. Also, hand in a listing of the m-file that you used for the evaluation of f and f'.

   Please don't waste lots of paper printing unnecessary things !

   (b) Modify `newton.m` so that implements the secant method (Lab Worksheet #2). Call the resulting code `secant.m` and hand in a listing of your code.

   (c) Using `secant.m`, find a root of f defined in (1). Discuss how your initial guesses for the iterations were chosen. Cut and paste the MATLAB output into a text document and hand that in.

   (d) Modify `newton.m` so that it implements the following iteration (recall from class that this method doesn't require two initial guesses, but is similar to the secant method):

$$x_{n+1} = x_n - \left(\frac{f(x_n)^2}{f(x_n + f(x_n)) - f(x_n)} \right).$$

   Hand in a listing of your code.

   (e) Compare the convergence of the above methods and of the bisection method by plotting the log of the step size $e_{n+1} = x_{n+1} - x_n$ versus iteration for each method **in the same figure**. For this you will need to use the plotting function `semilogy`, the `hold on` and `hold off` commands, and the output array `iter_hist`. In order to distinguish between the error curves, use different symbols for each error curve and give your figure a `legend`. Also, give your figure an appropriate `xlabel`, `ylabel`, and `title`. For a fair comparison, use the same initial guess x_0 for each of the above methods. For the bisection method, use this initial guess as one of your endpoints.

2. **Extra Credit:** The bisection method is said to have good global characteristics because provided you choose your initial interval so that it contains a root, you are guaranteed convergence to a root. Unfortunately, the bisection method has poor local characteristics because it is slow to converge. Newton's method, on the other hand, has poor global characteristics, because convergence is guaranteed only if the initial guess is "close enough" to a root. If the initial guess is close enough, though, Newton's method converges rapidly, and hence, we say that it has excellent local characteristics.

   Problem: Using the above information, create a hybrid method that has both good global and good local characteristics. Test your algorithm on (1) and compare your results with those obtained using Newton's method and the bisection method individually.