Math 221-QUIZ 2 NAME: Key

1. Reduce the following system to upper triangular form, circle the pivots, then solve the system using backwards substitution:

7

$$\begin{aligned} 2x - 3y + z &= 3 \\ 4x - 5y + z &= 1 \\ 2x + 2y - 3z &= 2. \end{aligned}$$

$-2r1+r2$ →

$$\begin{aligned} 2x - 3y + z &= 3 \\ y - z &= -5 \\ 2x + 2y - 3z &= 2 \end{aligned}$$

$-r1+r2$ →

$$\begin{aligned} 2x - 3y + z &= 3 \\ y - z &= -5 \\ 5y - 4z &= -1 \end{aligned}$$

$-5r2+r3$ →

$$\begin{aligned} 2x - 3y + z &= 3 \\ y - z &= -5 \\ z &= 24 \end{aligned}$$

pivots

$$x = \frac{1}{2}(3 + 3(19) - 24) = \frac{36}{2}$$

$$y = 19$$

$$z = 24$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 19 \\ 24 \end{bmatrix}$$

2. Let $\mathbf{A}$ be the coefficient matrix for the above system of equations. Which three matrices $\mathbf{E}_{21}$, $\mathbf{E}_{31}$ and $\mathbf{E}_{32}$ put $\mathbf{A}$ into triangular form $\mathbf{U}$? **Hint:** Use your work in Problem 1.

3

from above:

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 4 & -5 & 1 \\ 2 & 2 & -3 \end{bmatrix}, \quad \text{and} \quad \mathbf{E}_{32}\mathbf{E}_{31}\mathbf{E}_{21}\mathbf{A} = \mathbf{U}.$$

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \quad E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix}$$

(2x2 or 3x3)

Extra Credit: Write down three systems of equations satisfying the following properties: (i) one that has infinitely many solutions, (ii) one that has no solution, (iii) one that requires a row exchange to obtain a triangular system.

(i) ~~$x+x=5$~~ $\quad x+y=5$, $x+y=5$ $\xrightarrow{-r1+r2}$ $x+y=5$, $0y=0$ — infinitely many solutions

(ii) $x+y=5$, $x+y=2$ $\xrightarrow{-r1+r2}$ $x+y=5$, $0y=-3$ — no solutions

(iii) $x+y+z=1$, $x+y+2z=2$, $x+2y+2z=3$ $\xrightarrow[-r1+r3]{-r1+r2}$ $x+y+z=1$, $z=1$, $y+z=2$

$\xrightarrow{r2 \leftrightarrow r3}$ $x+y+z=1$, $y+z=2$, $z=1$