HW 2.7: 1, 2, 5, 6, 12, 20, 22

1. You do this

2. You do the computations.

Claim: $AB = BA$ implies $B^TA^T = A^TB^T$.

Proof: $(AB)^T = (BA)^T$ since $AB = BA$. Thus $B^TA^T = A^TB^T$. ■

5. a) $\begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix} = 5$

b) $\begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$

c) $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

6. $M^T = \begin{bmatrix} A^T & C^T \\ B^T & D^T \end{bmatrix}$

So $M^T = M$ if ~~A & D are symmetric~~ A, B, C, D are symmetric and $B = C$.

12.

$x \cdot y = x_1y_1 + x_2y_2 + \cdots + x_ny_n$

$Px \cdot Py$ = same thing w/ order of sum interchanged.

$= x_1y_1 + \cdots + x_ny_n = x \cdot y$

Thus $(Px)^T(Py) = x^TP^TPy$

$= x^T y$

Thus $P^TP = I$.

Let $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Then

$(Px) \cdot y = (2,3,1)\cdot(1,4,2) = 2+12+2 = 16$

$x \cdot (Py) = (1,2,3)\cdot(4,2,1) = 4+4+3 = 11$

20. $\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} \xrightarrow[\ell_{21}=3]{} \begin{bmatrix} 1 & 3 \\ 0 & -7 \end{bmatrix}$

$\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & -7 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -7 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$.

$= LDL^T$

$\begin{bmatrix} 1 & b \\ b & c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}\begin{bmatrix} 1 & b \\ 0 & c-b^2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & c-b^2 \end{bmatrix}\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$.

$= LDL^T$

$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} =$ [redacted] $\ell_{21} = -\frac{1}{2}$, $\ell_{31} = 0$, $\ell_{32} = -\frac{2}{3}$

25

$= \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 & 0 \\ 0 & 3/2 & -1 \\ 0 & 0 & 4/3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3/2 & 0 \\ 0 & 0 & 4/3 \end{bmatrix}\begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 1 & -2/3 \\ 0 & 0 & 1 \end{bmatrix}$

$= LDL^T$

22. $PA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$, $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

don't do $A = L_1P_1U_1$

$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 3 & 2 \end{bmatrix}$ $\ell_{21} = 0$, $\ell_{31} = 2$

$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ $\ell_{32} = 3$

So

$PA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

You do the other one (exchange rows 2 & 3)